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REACTOR MINIMUM CRITICAL DIMENSIONS

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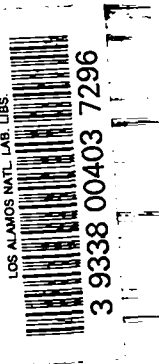
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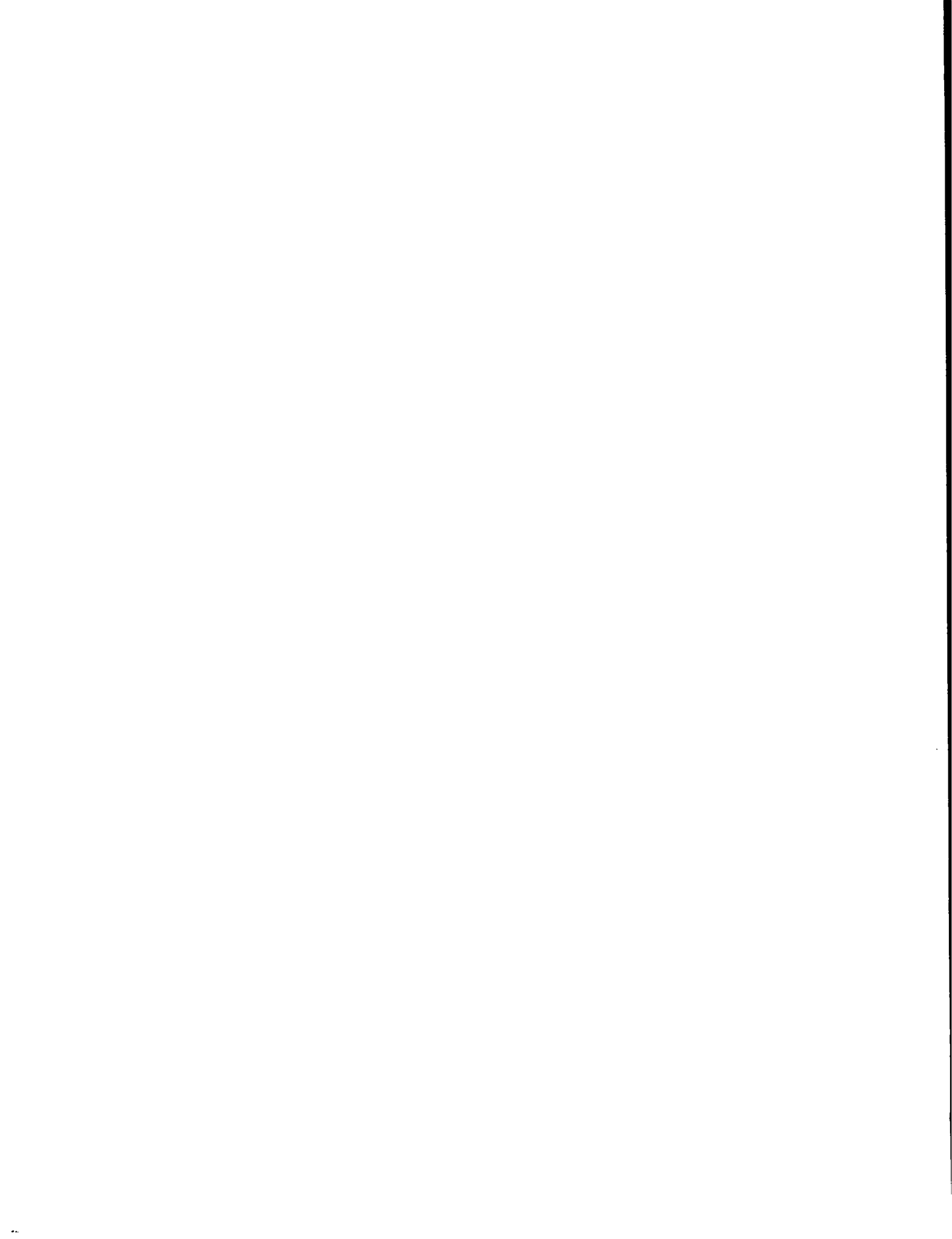
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REACTOR MINIMUM CRITICAL DIMENSIONS

by

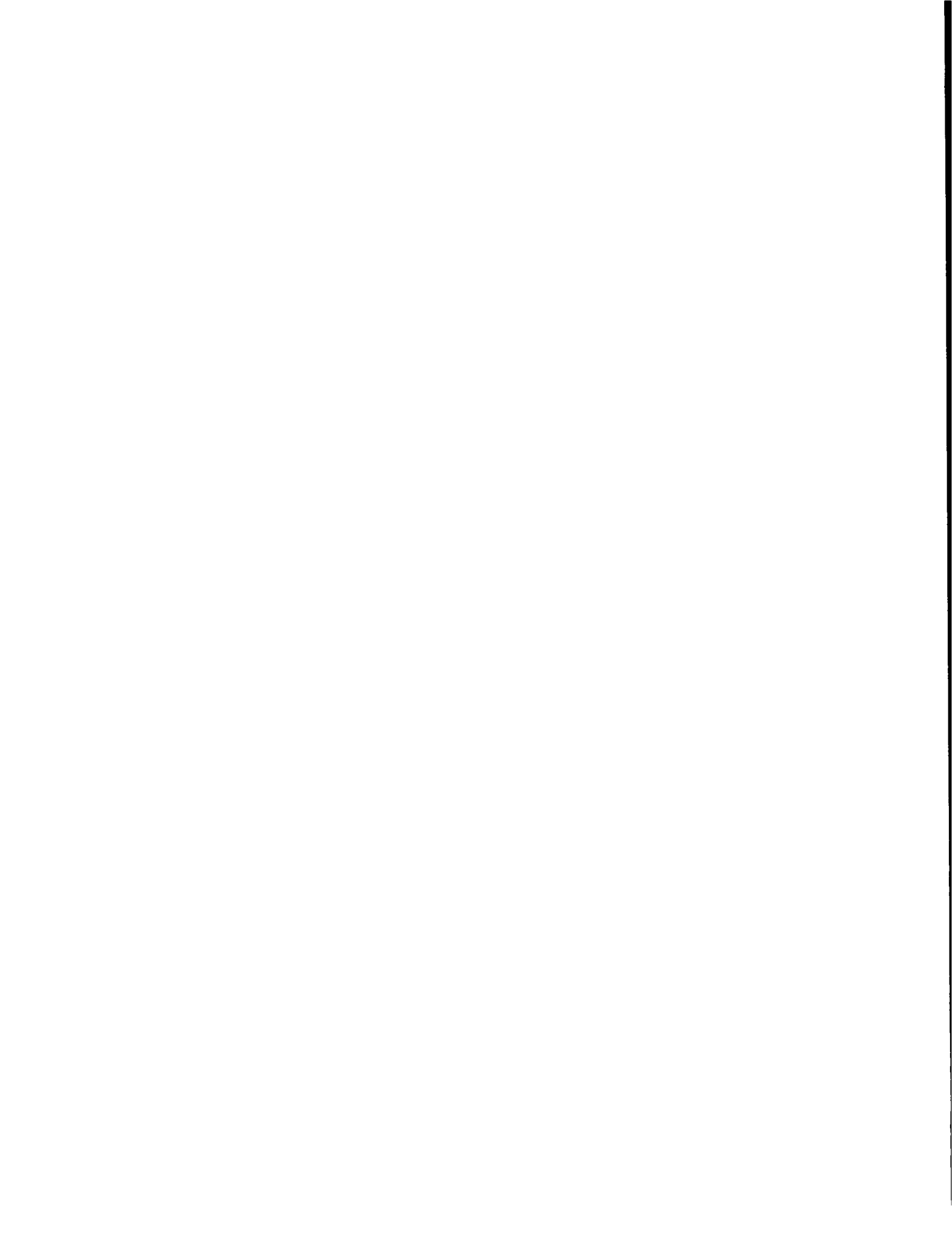
Carroll B. Mills





ABSTRACT

The parametric study of minimum critical reactor dimension as a function of moderator, fissionable isotope, and size has been made, based on a consistent variety of critical experiments studied in a companion report (LA-3219-MS). Minimum critical size and mass have been computed for a range of concentration of U^{233} , U^{235} , and Pu^{239} for H_2O -moderated bare and reflected slab, cylinder, and sphere geometries, as well as corresponding results for U^{235} and heavier atom moderators D_2O , Be, BeO, and C. Some results are presented of the same sort for D_2O , Be, and C reflector-moderated reactors.



INTRODUCTION

A knowledge of what to expect in reactor criticality is often useful, both for qualitative studies and for reactor safety studies.

The elements of a critical parameter survey are:

1. A numerical transport approximation which will include the effects of neutron flux anisotropies near boundaries.
2. A complete set of group-averaged transport and absorption cross sections, including fission-neutron energy spectrum and production per fission absorption as a function of energy.
3. Sufficient evaluation of reference critical experiments so that the homogeneities and other things not effecting a pure, homogeneous critical assembly can be eliminated.

HYDROGEN-MODERATED, ENRICHED, FISSIONABLE MATERIAL SYSTEMS

The survey of H₂O-moderated and reflected slab, cylinder, or sphere geometries containing U²³³, U²³⁵, or Pu²³⁹ assume atomic displacement of water by the fuel:

$$N(\text{U}^{233} \text{ or } \text{U}^{235}) = 0.0480 (1 + 0.720 \text{ H/U})^{-1}$$

$$N(\text{Pu}^{239}) = 0.0494 (1 + 0.741 \text{ H/Pu})^{-1}$$

where N is atomic density in units of 10^{24} atoms/cc and H/U or H/Pu is the atomic ratio of hydrogen to fuel atom. The corresponding hydrogen atomic density $N(\text{H})$ is $(\text{H/U})N(\text{U})$, and that for oxygen is $N(\text{H})/2$. These formulas

presume densities of pure materials to be $\rho(U^{235}) = 18.75$ g/cc and $\rho(Pu^{239}, \alpha\text{-phase}) = 19.6$ g/cc.

Numerical calculation of critical radii or thickness, and from this critical mass and volume for spheres, were made for H/U ratios 0, 1, 3, 10, ..., 3000 for H₂O solutions. Results are presented in Table 1 for dimensions only and on accompanying Figs. 1 to 12 for mass and volume.

HEAVIER ATOM MODERATORS

Parametric studies of the other moderators, D₂O, Be, BeO, and C, are somewhat more difficult to validate than those with H₂O because of the large effects of H₂O impurity in D₂O, thick U²³⁵ fuel foils in Be and BeO, and an uncertainty in B content in the C-moderated reactors. Studies of the same sort, but for fewer geometries, are made as shown below, using atomic density formulas:

$$\begin{aligned} N(U^{235}) \text{ in } D_2O &= 0.0480 (1 + 0.720 D/U)^{-1} \\ N(U^{235}) \text{ in Be} &= 0.0480 (1 + 0.404 Be/U)^{-1} \\ N(U^{235}) \text{ in BeO} &= 0.0480 (1 + 0.7023 BeO/U)^{-1} \\ N(U^{235}) \text{ in C} &= 0.0480 (1 + 0.582 C/U)^{-1} \\ N(Pu^{239}) \text{ in C} &= 0.04935 (1 + 0.5983 C/Pu)^{-1} \end{aligned}$$

which assume for densities in grams per cubic centimeter:

$$\begin{aligned} \rho(D_2O) &= (H_2O) \times 1.1116 & \rho(U^{235}) &= 18.75 \\ \rho(Be) &= 1.78 & \rho(Pu^{239}) &= 19.6 \\ \rho(BeO) &= 2.84 \\ \rho(C) &= 1.645 \end{aligned}$$

TABLE 1. CRITICAL CORE RADIUS (cm) * OF H₂O SOLUTIONS †

Atomic ratio H/U	‡ ρ, kg/l	U ²³³			U ²³⁵			Pu ²³⁹		
		Infinite slab	Infinite cylinder	Sphere	Infinite slab	Infinite cylinder	Sphere	Infinite slab	Infinite cylinder	Sphere
<u>A. Bare</u>										
0	18.740	4.3	4.0	5.8	6.62	5.93	8.41	3.70	3.47	5.05
1	10.870	5.82	5.2	7.43	8.52	7.43	10.42	5.44	5.15	7.00
3	5.920	7.2	6.3	8.83	10.56	9.04	12.58	7.80	6.90	9.30
10	2.284	10.2	7.9	10.8	12.00	10.20	14.30	10.42	9.10	12.26
30	0.8288	10.4	8.72	12.0	12.70	10.80	14.50	12.00	10.20	13.90
100	0.2567	11.4	9.6	13.1	12.90	11.20	14.70	12.90	10.70	14.70
300	0.0864	13.1	10.9	15.0	14.20	12.20	16.70	14.00	11.70	15.70
500	0.0520	14.9	12.4	17.0	16.20	13.80	18.80	15.10	12.60	16.90
1000	0.02599	20.8	16.8	22.6	23.20	18.90	25.10	18.42	15.10	20.20
1500	0.01733	30.4	24.2	32.4	35.40	28.50	38.00	23.10	18.70	24.90
2000	0.01288	56.4	47.0	63.1	71.80	57.80	77.70	28.90	22.80	30.91
3000	0.00867							60.70	51.60	69.51
<u>B. Reflected</u>										
0	18.740	0.60	2.2	4.1	1.8	3.45	6.00	0.83	2.00	3.81
1	10.870	0.75	2.82	5.1	2.47	4.30	7.21	1.33	3.04	5.26
3	5.920	1.36	3.8	6.1	3.30	5.20	9.00	2.08	4.38	7.05
10	2.284	2.2	4.8	7.6	4.04	6.40	10.43	3.23	5.96	8.83
30	0.8288	2.9	5.50	8.9	4.60	6.77	10.66	4.14	6.75	10.05
100	0.2567	4.0	5.9	9.5	5.08	6.96	10.99	5.08	6.97	10.86

TABLE 1 (continued)

Atomic Ratio H/U	# $\rho, \text{kg/l}$	U^{233}			U^{235}			Pu^{239}		
		Infinite slab	Infinite cylinder	Sphere	Infinite slab	Infinite cylinder	Sphere	Infinite slab	Infinite cylinder	Sphere
300	0.0864	6.1	7.1	11.3	6.90	8.60	13.00	6.40	7.80	12.10
500	0.0520	8.1	8.6	13.4	8.90	9.90	15.20	7.50	8.70	13.40
1000	0.0260	13.8	13.4	19.1	16.30	14.99	21.44	10.60	11.33	16.67
1500	0.01733	23.8	20.3	28.7	29.60	24.80	34.66	15.00	14.80	21.30
2000	0.01288	49.0	43.5	59.5	64.00	54.00	74.10	21.10	18.77	27.41
3000	0.0867							52.90	50.40	66.00

* Slab thickness.

+ Limiting concentrations: U^{233} : 0.01159; U^{235} : 0.01166; and Pu^{239} : 0.00778

Density of U^{235} . Multiply by 0.9915 for U^{233} ; by 1.0816 for Pu^{239} .

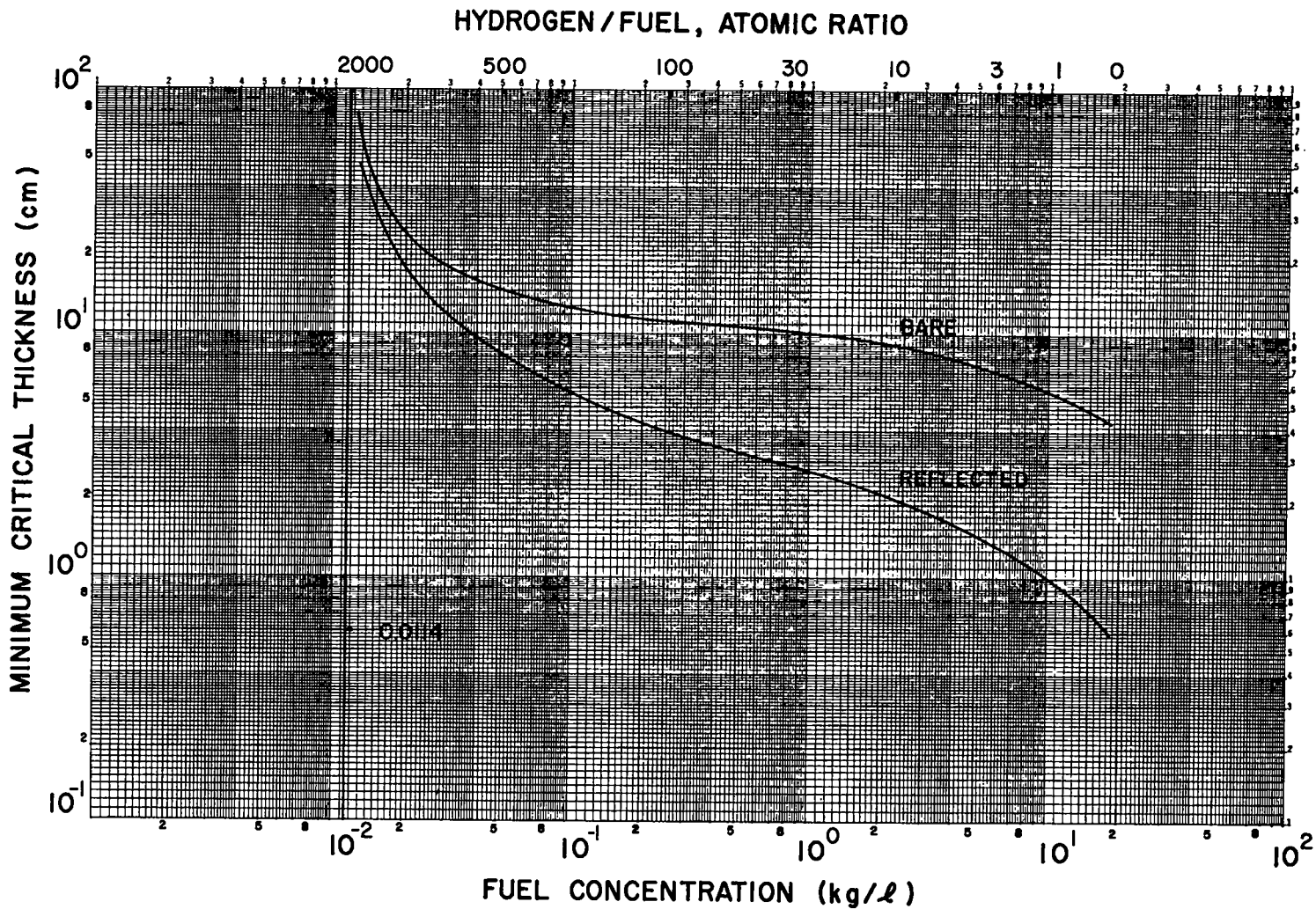


Fig. 1. Minimum critical thickness of a bare and water-reflected slab of U^{233} - H_2O solution.

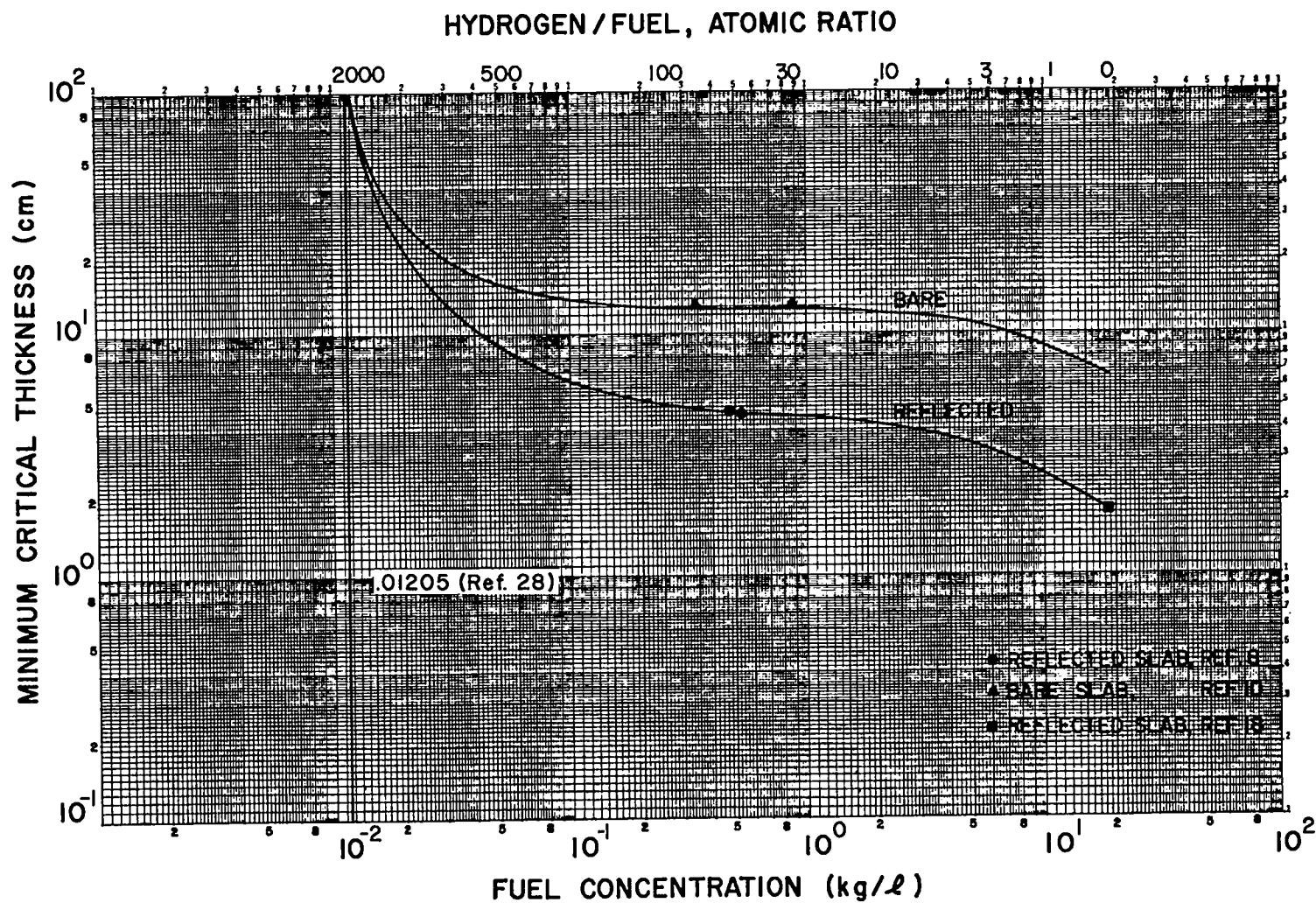


Fig. 2. Minimum critical thickness of a bare and water-reflected slab of U^{235} - H_2O solution.

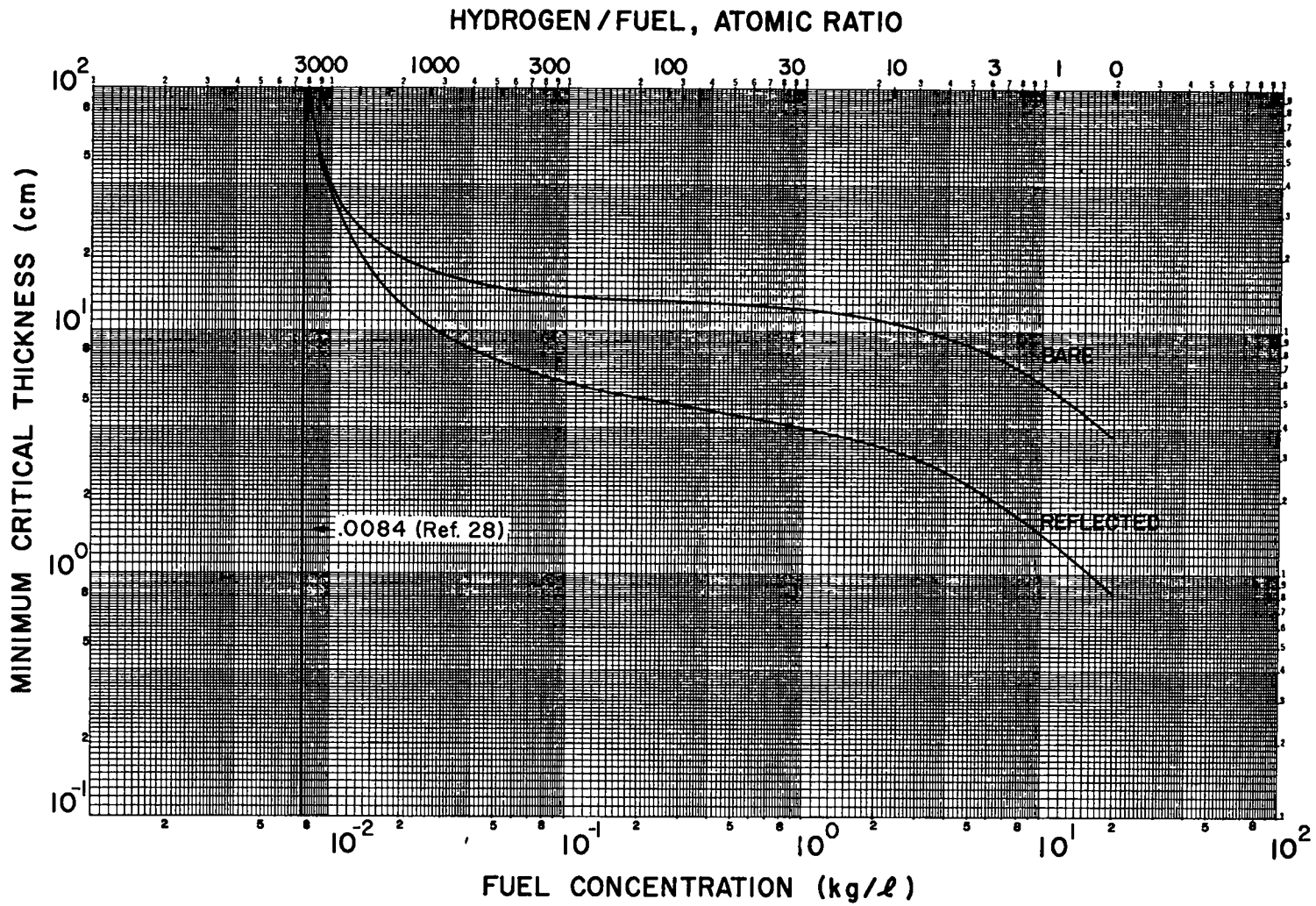


Fig. 3. Minimum critical thickness of a bare and water-reflected slab of $\text{Pu}^{239}\text{-H}_2\text{O}$ solution.

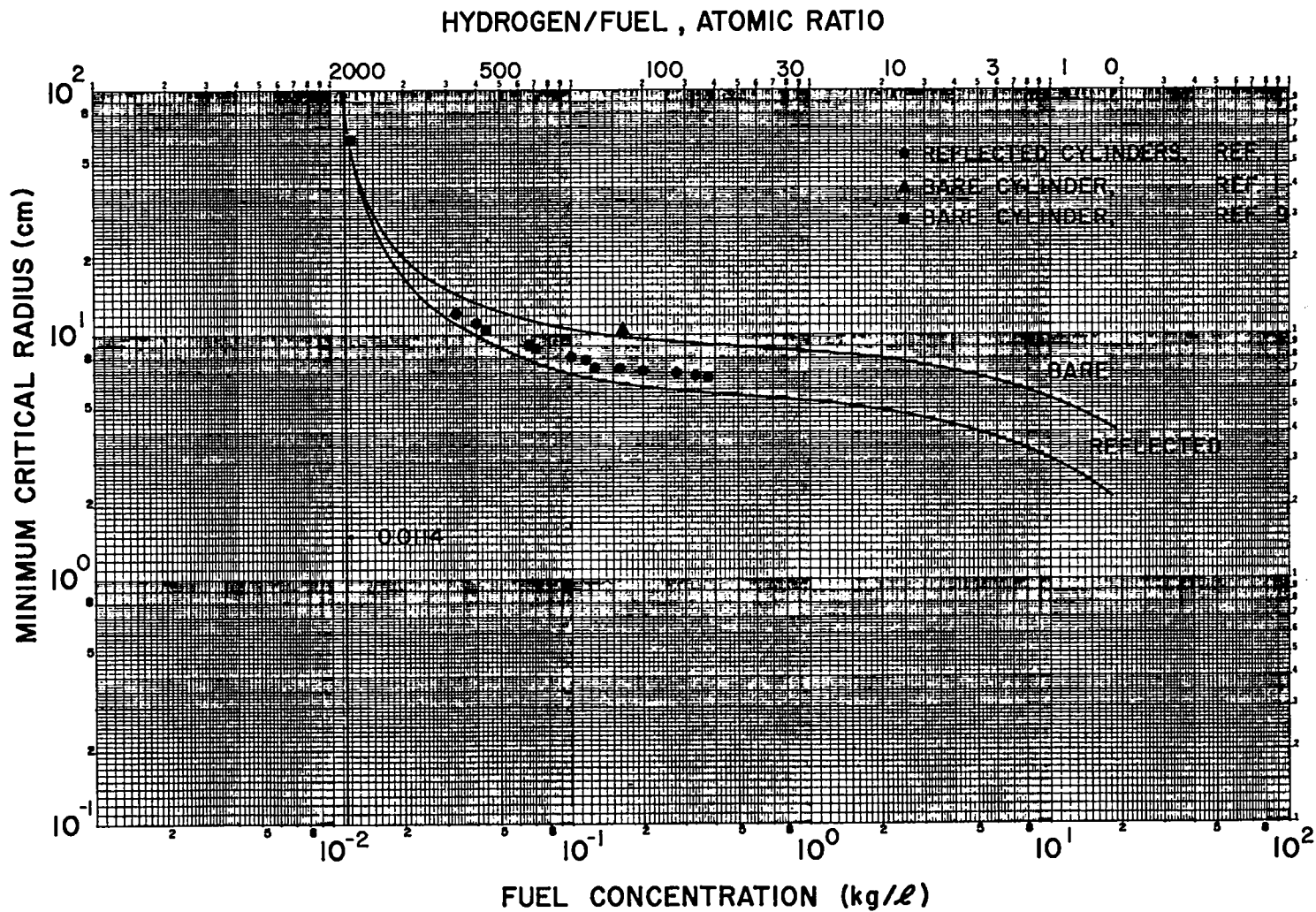


Fig. 4. Minimum critical radius of a bare and water-reflected cylinder of U^{233} - H_2O solution.

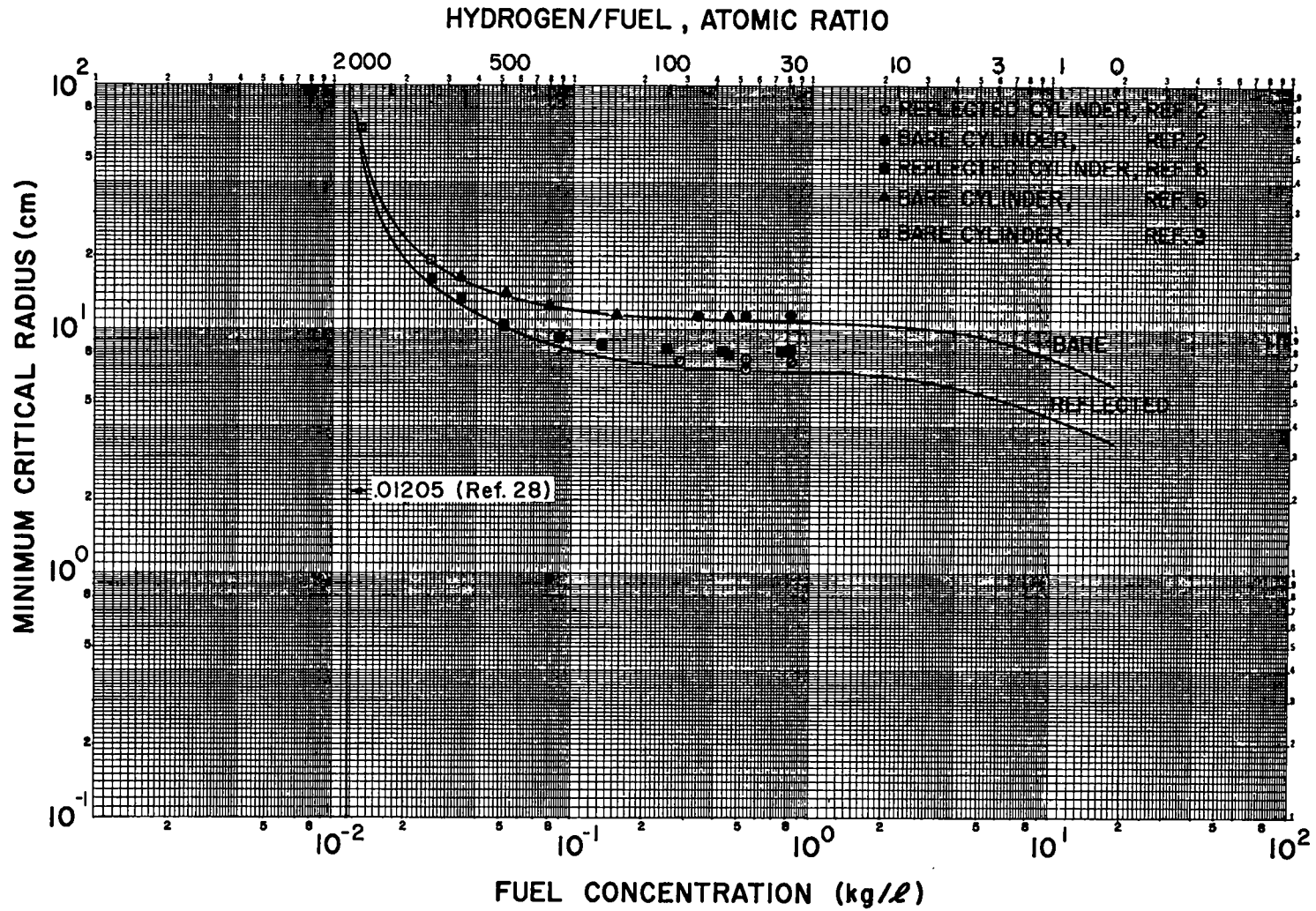


Fig. 5. Minimum critical radius of a bare and water-reflected cylinder of U^{235} - H_2O solution.

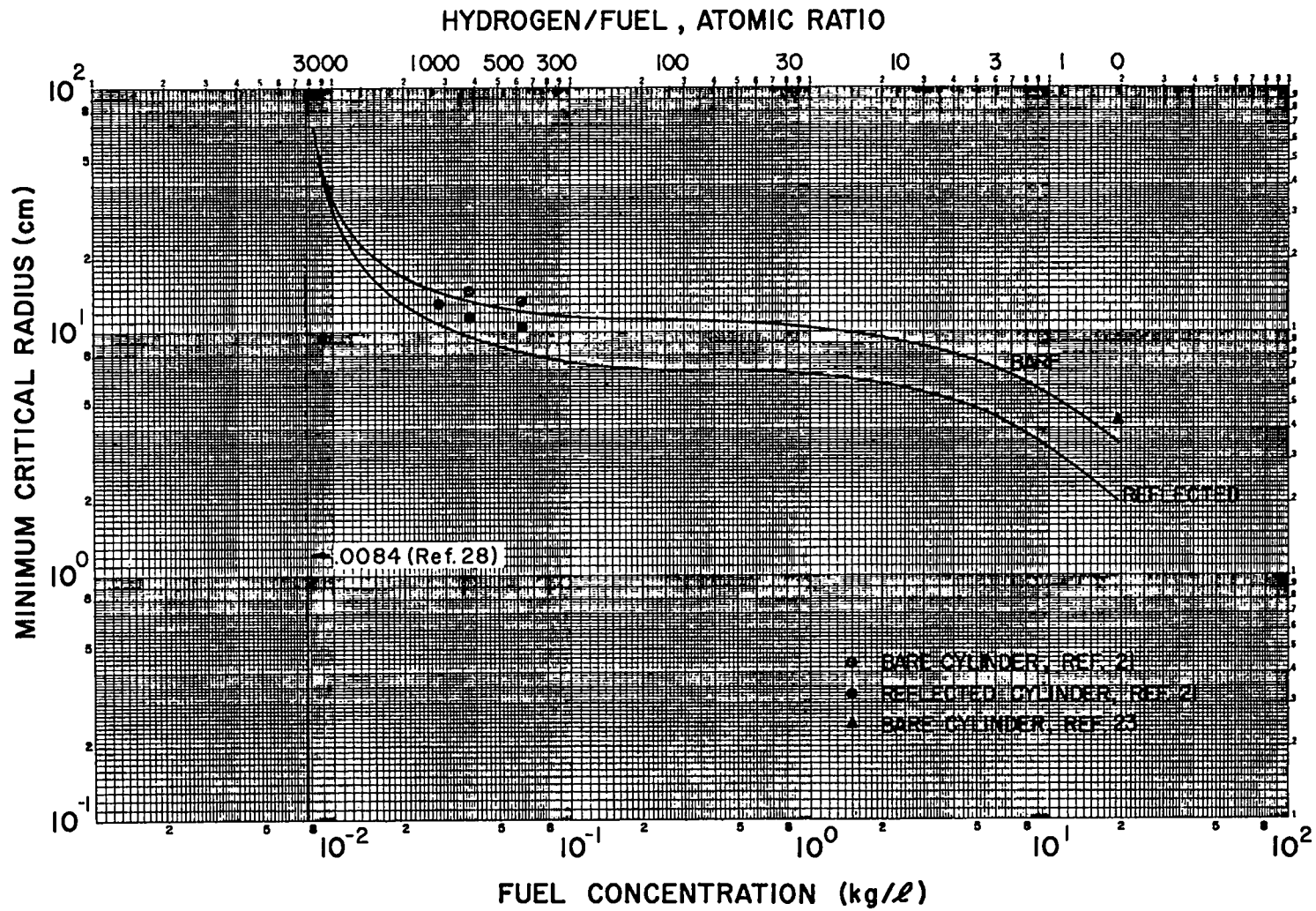


Fig. 6. Minimum critical radius of a bare and water-reflected cylinder of $\text{Pu}^{239}\text{-H}_2\text{O}$ solution.

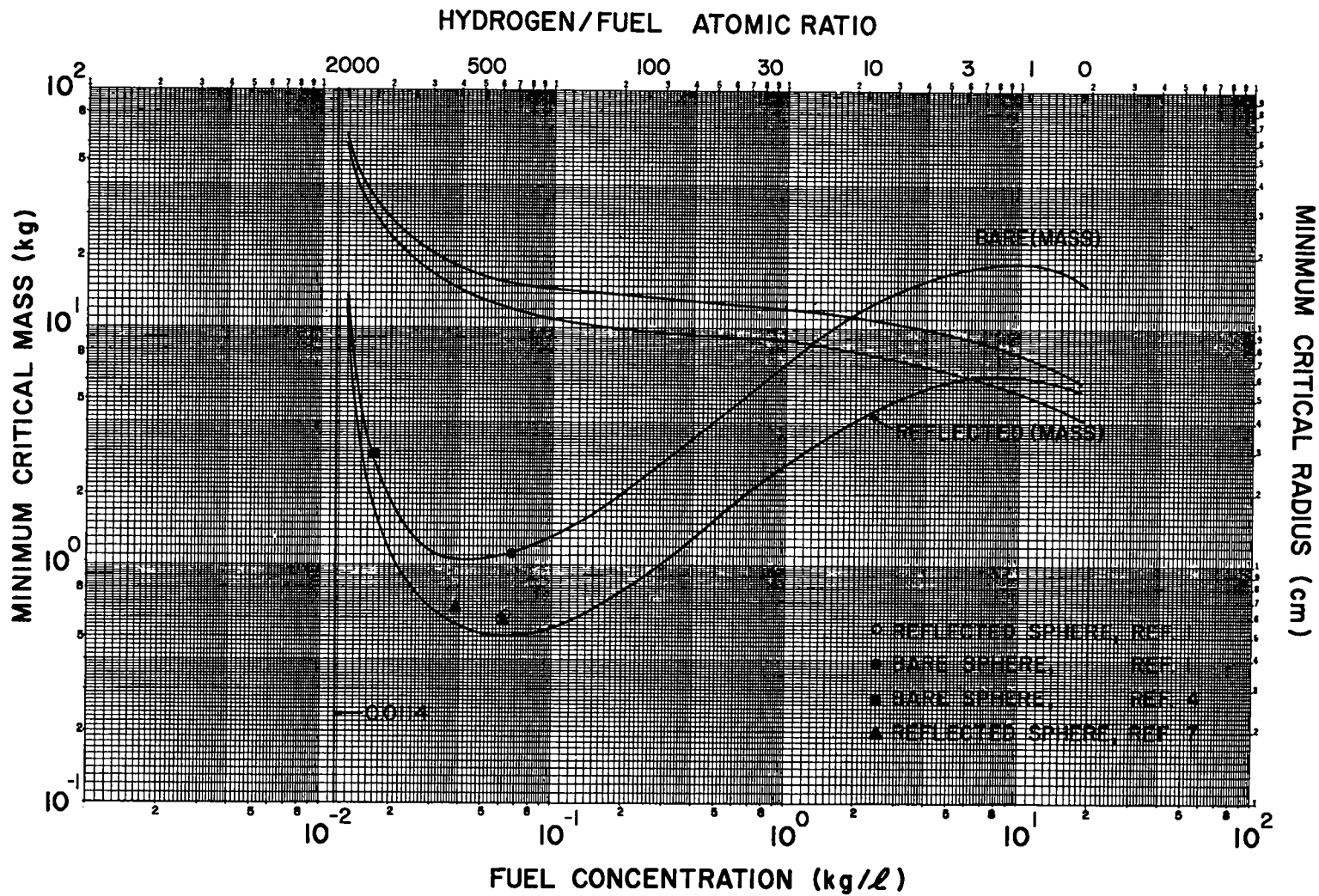


Fig. 7. Minimum critical mass of a bare and water-reflected sphere of U^{233} - H_2O solution.

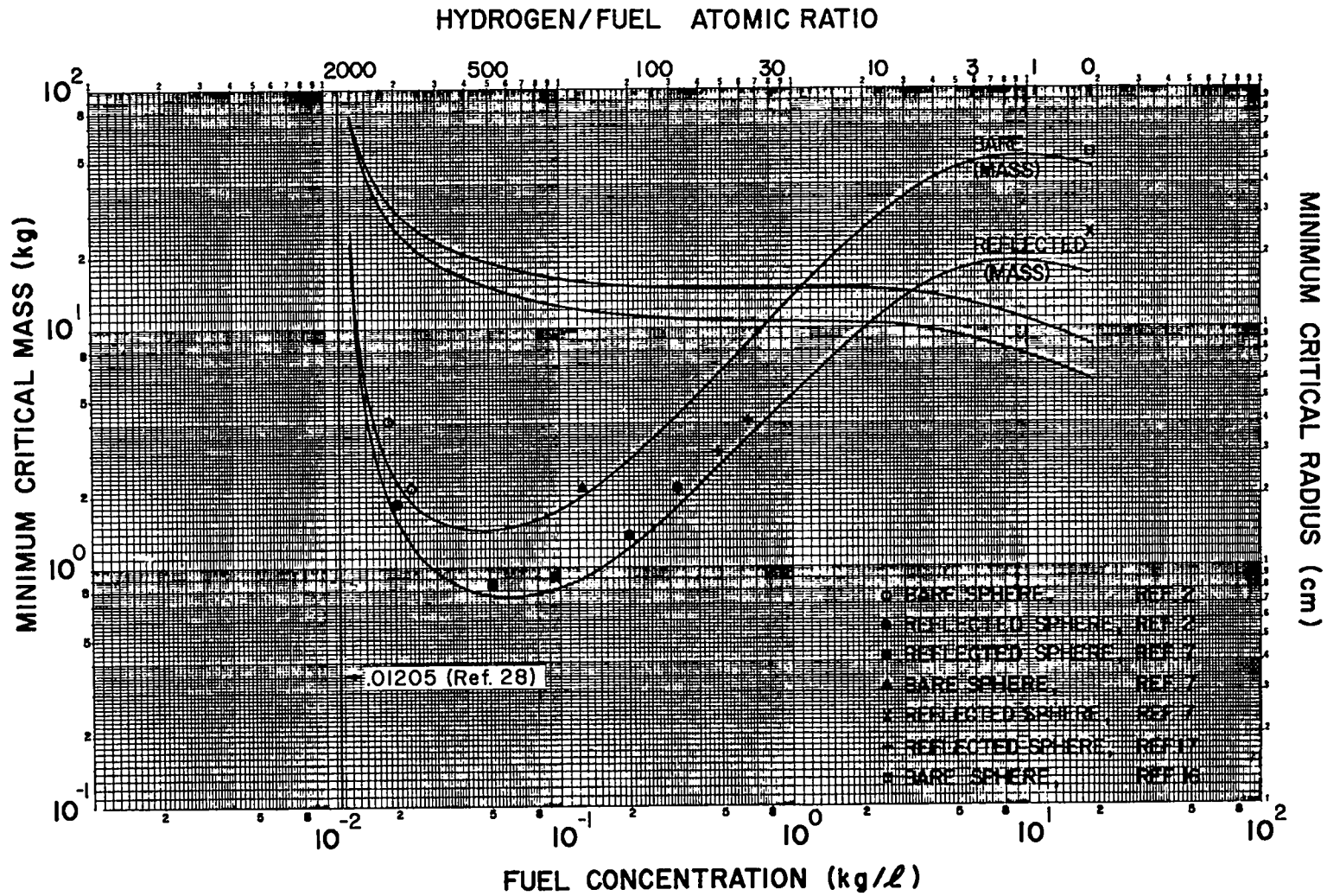


Fig. 8. Minimum critical mass of a bare and water-reflected sphere of U^{235} - H_2O solution.

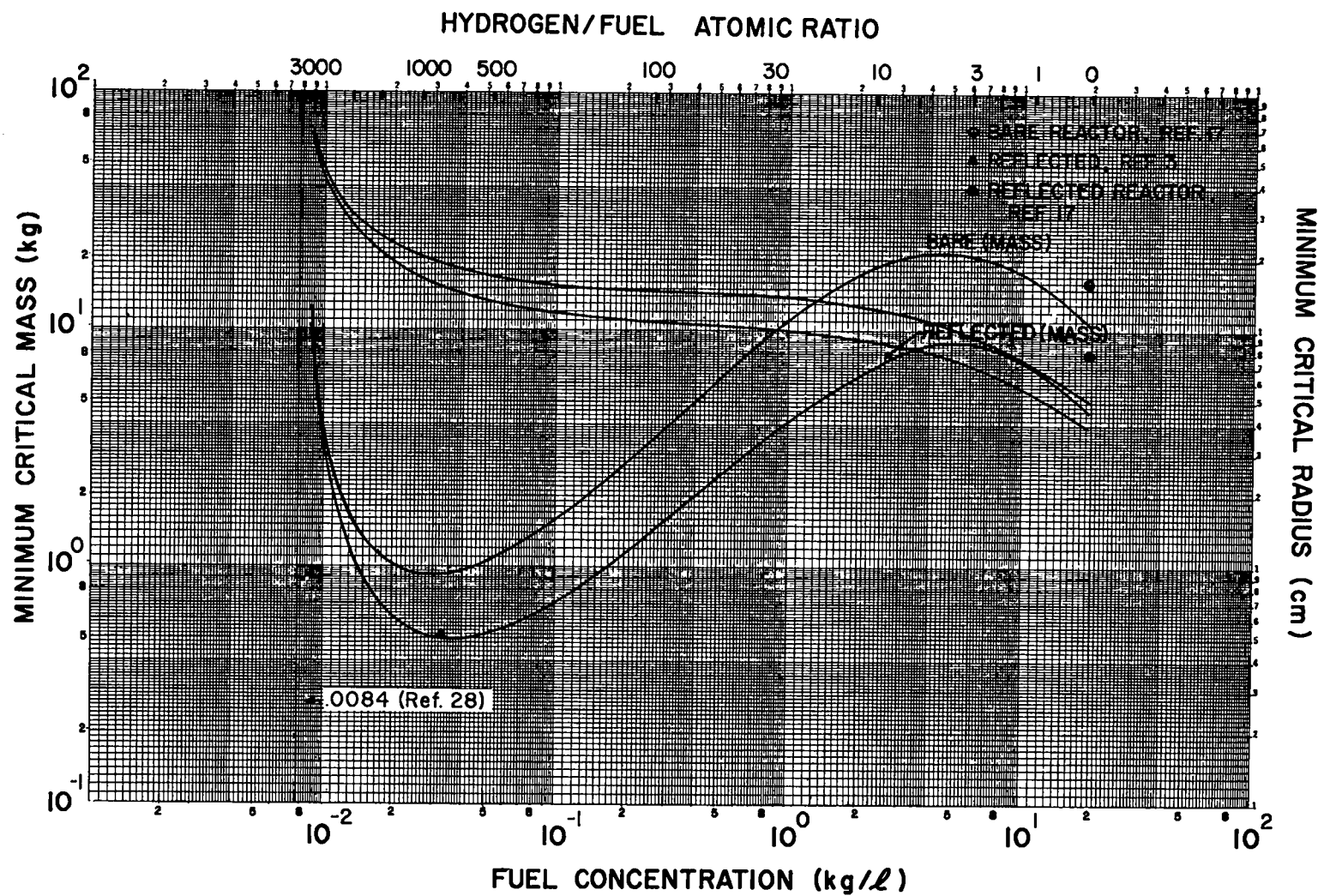


Fig. 9. Minimum critical mass of a bare and water-reflected sphere of $\text{Pu}^{239}\text{-H}_2\text{O}$ solution.

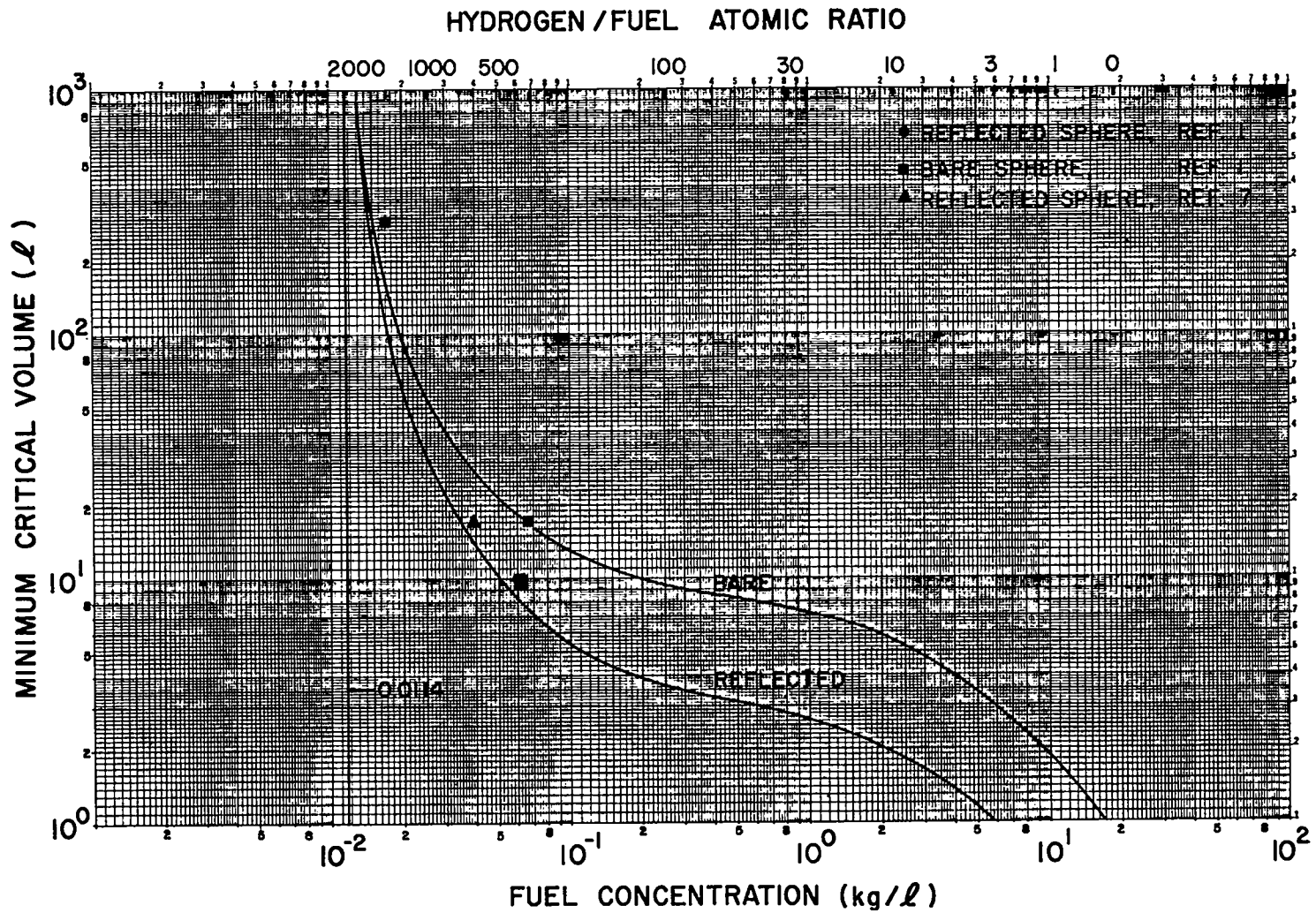


Fig. 10. Minimum critical volume of a bare and water-reflected sphere of U^{233} - H_2O solution.

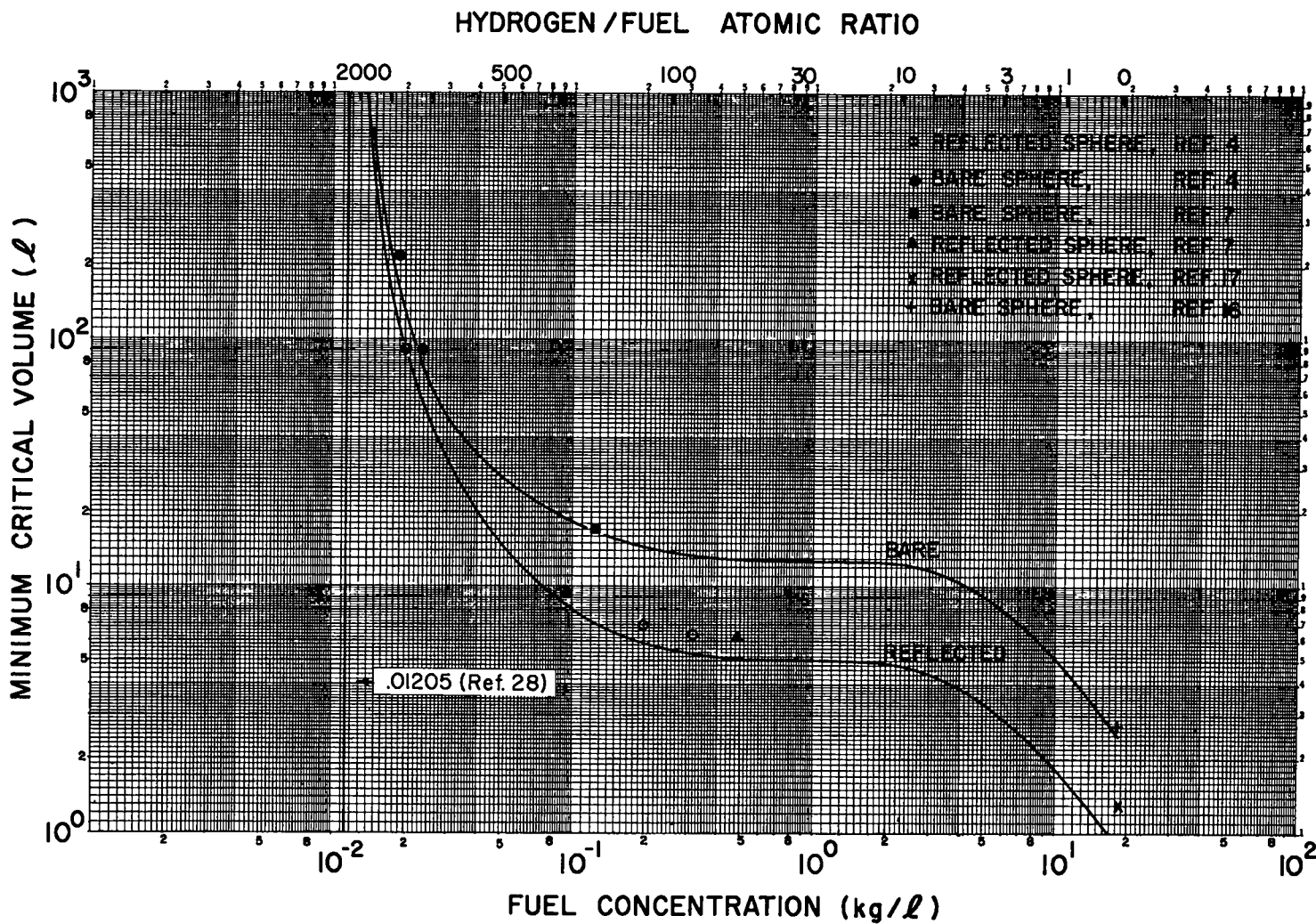


Fig. 11. Minimum critical volume of a bare and water-reflected sphere of U^{235} - H_2O solution.

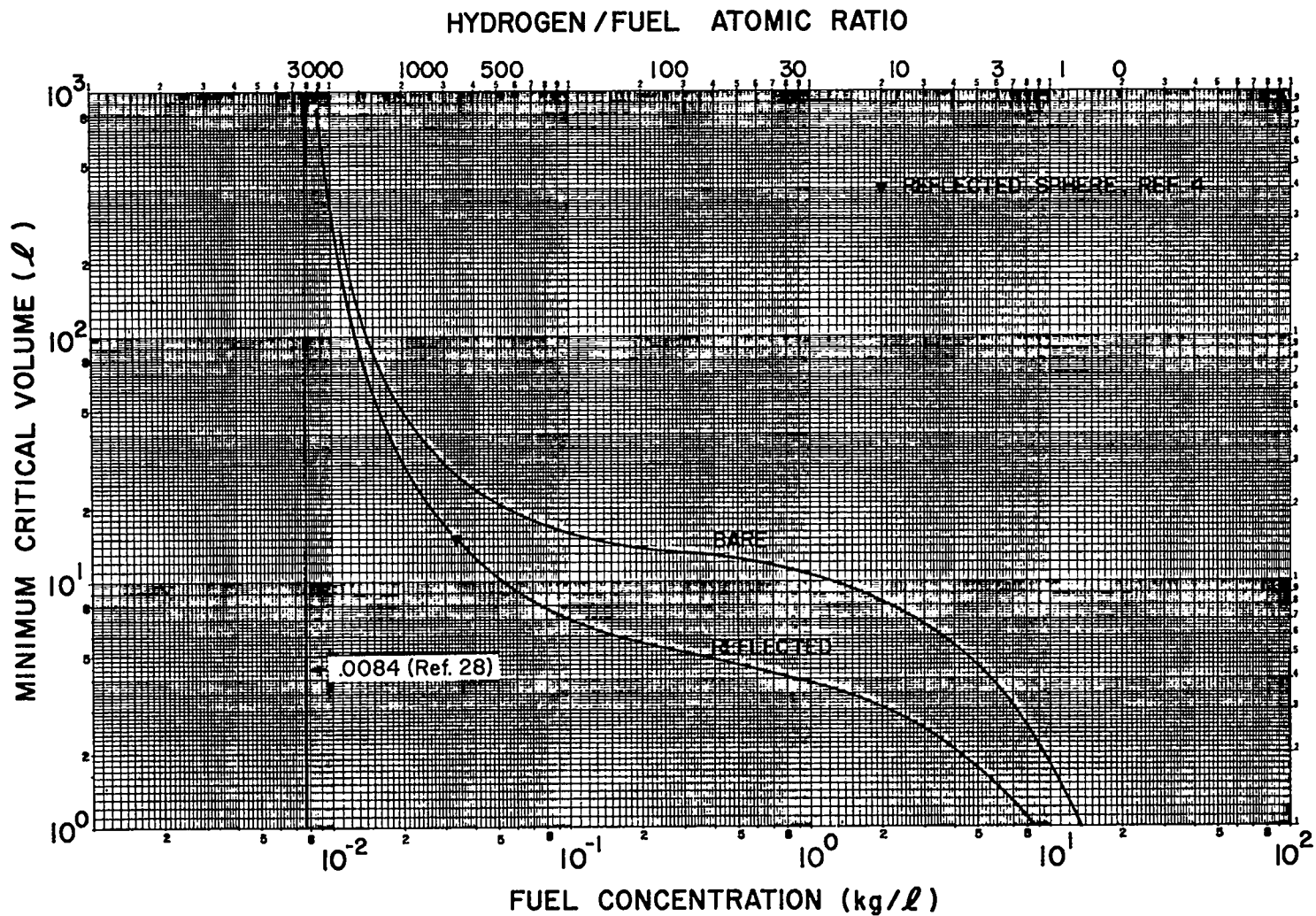


Fig. 12. Minimum critical volume of a bare and water reflected sphere of $\text{Pu}^{239}\text{-H}_2\text{O}$ solution.

Spherical geometries for the above were studied, and the results are shown on Tables 2 and 3 and Figs. 13 and 14. The computed points are shown on the curves to permit separation of the several moderator/fuel ratios. Note that D_2O/U^{235} ratio points only are indicated on the graph abscissa.

As with the H_2O -moderated reactor study, the critical radii were computed in a manner consistent with the reference critical experiments. In other words, the complex experiments reduced to pure homogeneous spheres would agree with the graphical results shown above. While the experiments are very sparse and restricted in character, there is sufficient supplementary data in the form of neutron flux-dependent measurements that the study is expected to be quantitative. These aspects are discussed separately under analysis of critical experiments.

LOW-ENRICHMENT, HYDROGEN-MODERATED SYSTEMS

The relatively small number of reactor experiments using low-enrichment U^{235} in uranium, along with the wide interest for safety and power uses of low-enrichment uranium, emphasize the importance of parametric calculation. The following parametric survey, Table 4, for 1.4 to 30% U^{235} in uranium (by number) is based on experiments by Mihalczko and Cronin shown in Table 7 of LA-3219-MS. Resonance analyses by Bell permitted the direct quantitative calculation of critical radius and mass with appropriate treatment of U^{238} resonances in the wide range of conditions affecting

TABLE 2. CRITICALITY OF U^{235} IN D_2O , Be and BeO-MODERATED SPHERES

Atomic ratio moderator/ U^{235}	D_2O			Be			BeO		
	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l
1	12.2	82.7	10.87	10.7	68.5	13.35	11.5	70.1	11.01
3	16.0	100.8	5.92	13.4	85.4	8.47	14.6	78.7	6.03
10	22.4	107.6	2.29	17.4	82.1	3.72	20.2	80.7	2.34
30	29.2	86.5	0.829	22.0	64.6	1.43	26.3	64.7	0.849
100	34.8	45.3	0.257	27.4	39.0	0.453	31.5	34.5	0.263
300	36.7	17.89	0.0864	29.5	16.5	0.153	33.6	14.07	0.0885
1000	39.5	5.71	0.0260	30.4	5.44	0.0463	35.3	4.91	0.0267
3000	45.8	3.49	0.00867	33.2	2.37	0.0155	39.8	2.35	0.00889

TABLE 3. CRITICALITY OF U^{233} , U^{235} , AND Pu^{239} IN GRAPHITE-MODERATED SPHERES

Atomic ratio C/X	U^{233}			U^{235}			Pu^{239}		
	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l	Critical radius, cm	Critical mass, kg	Critical concentra- tion, kg/l
1	9.1	37.1	11.75	12.4	94.6	11.85	8.1	27.3	12.26
3	13.1	63.7	6.77	17.5	153.2	6.83	12.1	51.7	7.01
10	21.9	119.9	2.73	27.6	242.1	2.75	21.3	113.6	2.81
30	32.2	140.8	1.007	40.4	280.6	1.016	34.0	170.2	1.034
100	42.8	103.1	0.314	54.5	214.8	0.317	49.4	162.7	0.322
300	50.8	58.2	0.106	65.3	124.5	0.1068	60.4	100.2	0.1086
1000	57.3	25.1	0.0319	67.2	40.9	0.0322	67.0	41.2	0.0327
3000	63.8	11.58	0.01064	70.5	15.75	0.01073	70.2	15.8	0.01091
10000	76.4	5.96	0.00319	81.2	7.23	0.00322	76.5	6.14	0.00328

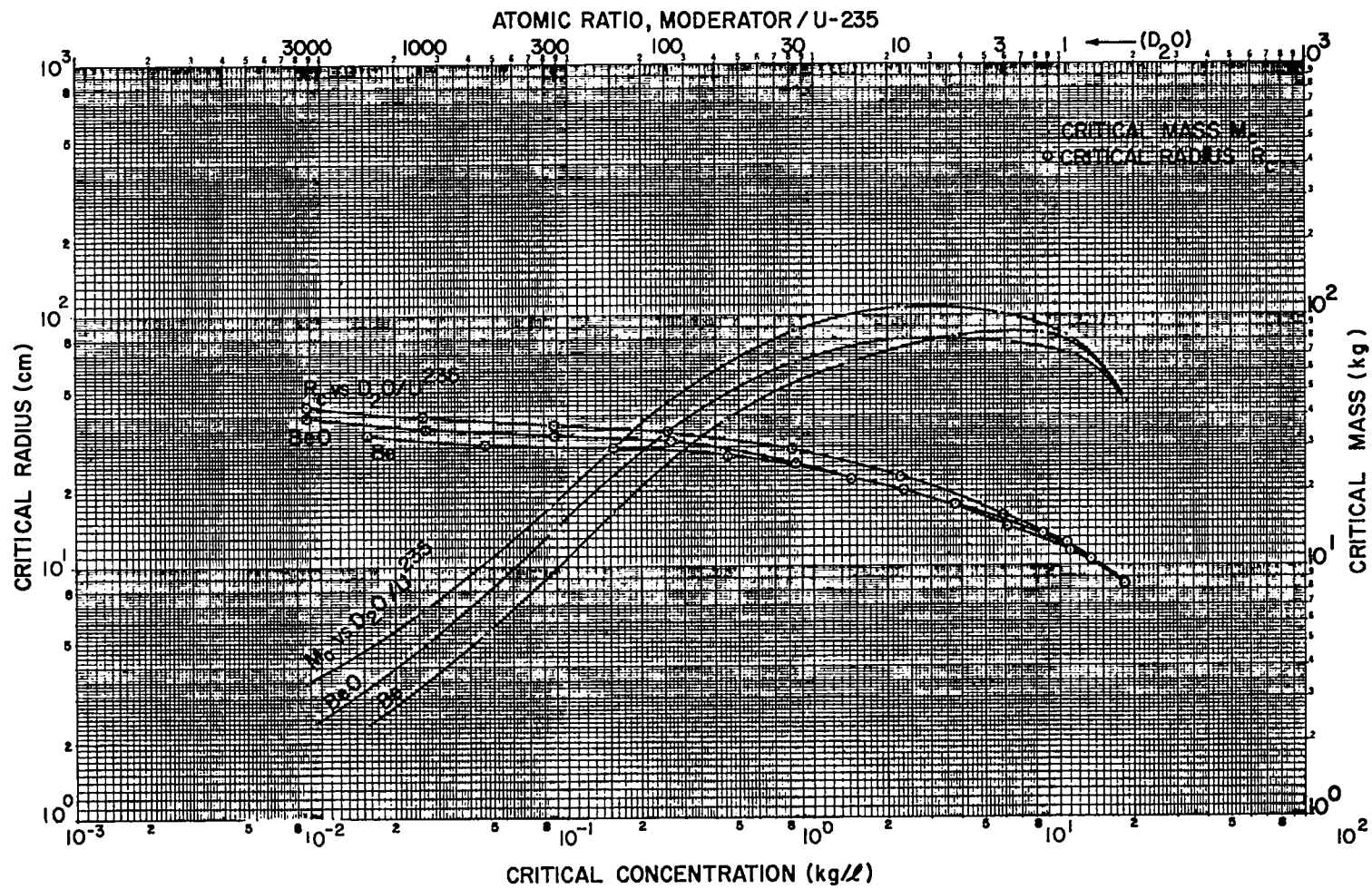


Fig. 13. Critical radius and mass vs U^{235} concentration for D_2O , Be, and BeO spheres.

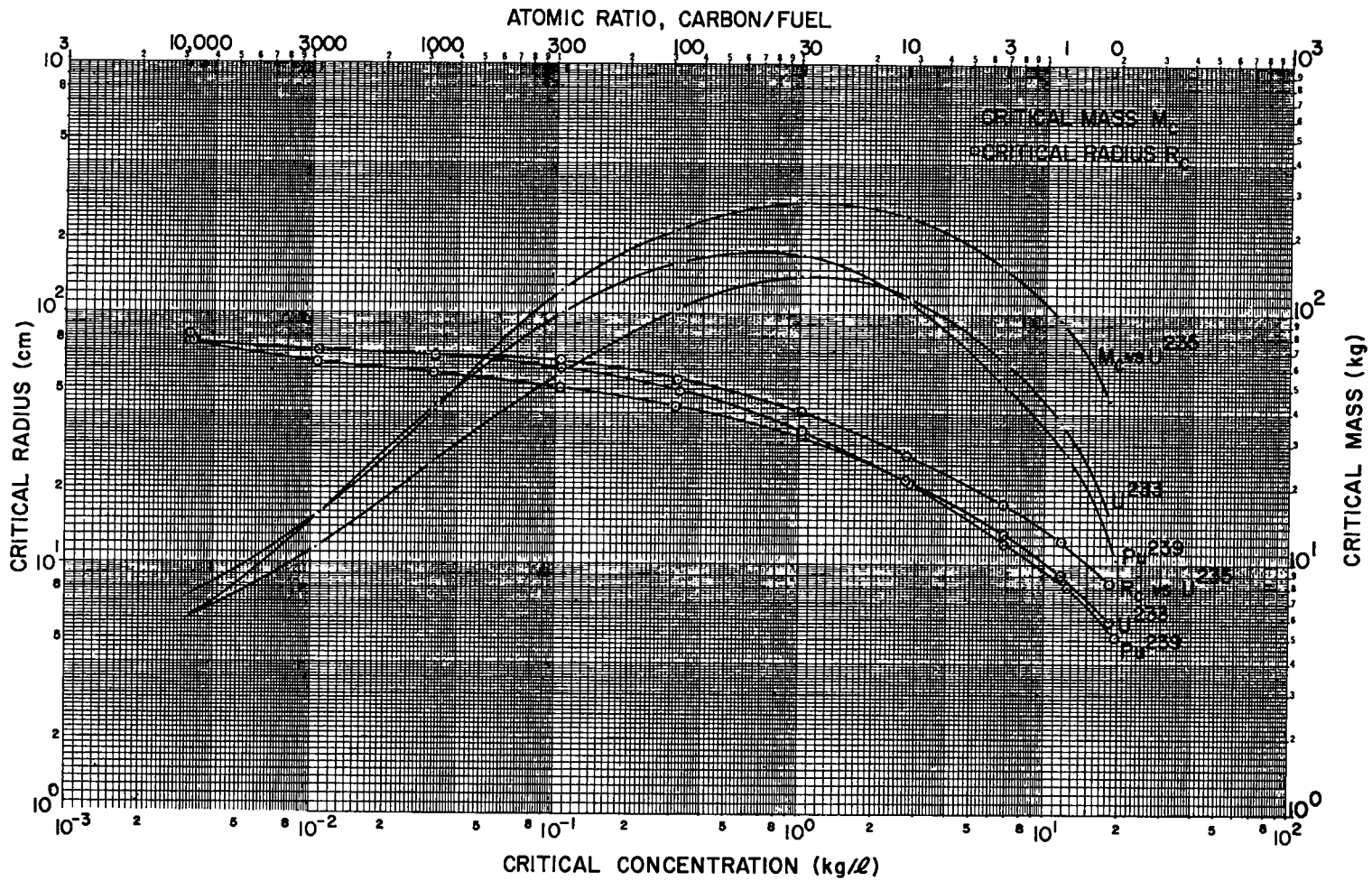


Fig. 14. Critical radius and mass of graphite spheres vs concentration of U^{233} , U^{235} , and Pu^{239} .

TABLE 4. CRITICAL BARE METAL-WATER SPHERES

U^{235} enrichment	Atomic ratio H/U^{235}	Density of uranium, gm/cc	Hydrogen atomic density, $\times 10^{24}$ atoms/cc	Critical radius, cm	Critical mass U^{235} , kg	Reflector savings, cm
0.014	127.50	8.299	0.03748	87.01	316.80	9.2
	255.00	5.134	0.04799	47.68	33.36	4.95
	320.00	4.501	0.05082	46.05	25.39	4.27
	478.00	3.260	0.05520	46.99	19.60	-
	638.00	2.555	0.05769	54.66	24.18	4.37
	956.00	1.739	0.06040	143.53	305.70	4.49
0.017	65.41	10.54	0.02965	201.50	6018.00	
	104.60	8.315	0.03743	55.49	100.10	
	261.90	4.513	0.05080	36.61	15.57	
	524.00	2.561	0.05766	38.68	10.41	
	785.00	1.787	0.06038	50.83	16.52	
	1047.00	1.373	0.06184	85.94	61.32	
0.020	55.40	10.55	0.2958	76.27	387.50	8.40
	88.70	8.33	0.03740	46.03	67.26	6.83
	221.80	4.525	0.05708	32.26	12.58	4.56
	443.40	2.568	0.05764	32.68	7.42	4.23
	665.50	1.792	0.06036	38.61	8.54	3.67
	887.00	1.377	0.06182	48.30	12.85	3.29
	1330.00	0.5869	0.06335	600.60	1686.70	-
0.030	36.59	10.33	0.02943	46.83	136.70	
	58.57	8.374	0.03722	35.10	44.97	
	146.40	4.556	0.05062	26.49	10.52	
	292.80	2.590	0.05755	25.47	5.31	
	439.30	1.8084	0.06029	27.24	5.54	
	585.60	1.3895	0.06175	29.32	4.35	
	1025.00	0.8200	0.06379	41.01	7.023	
	1464.00	0.5813	0.06458	64.05	18.97	

TABLE 4 (continued)

U^{235} enrichment	Atomic ratio H/U^{235}	Density of uranium, gm/cc	Hydrogen atomic density $\times 10^{24}$ atoms/cc	Critical radius, cm	Critical mass U^{235} , kg	Reflector savings, cm
0.050	8.62	14.480	0.01578	66.48	880.50	8.39
	34.46	8.465	0.03691	28.83	41.98	6.36
	86.18	4.624	0.05042	22.95	11.567	5.03
	172.30	2.633	0.05742	21.43	5.36	4.37
	258.60	1.840	0.06020	21.83	3.96	4.16
	344.70	1.415	0.06170	22.42	3.30	3.87
	603.00	0.8353	0.06373	25.17	2.756	-
	861.80	0.5924	0.06459	29.65	3.20	3.26
0.075	0	18.98	0	58.27	1165.00	-
	0.20	18.85	0.000216	61.56	1365.00	
	0.561	18.41	0.00196	70.53	2005.00	
	5.61	14.56	0.01550	38.24	252.70	
	22.45	8.576	0.03654	25.84	45.95	
	56.11	4.707	0.05012	21.23	13.98	
	112.20	2.687	0.06723	19.72	6.40	
	168.40	1.879	0.06006	19.68	4.45	
	561.30	0.6062	0.06455	22.94	2.27	
	1122.30	0.3079	0.06559	33.30	3.53	
	1400.00	0.2476	0.06579	44.45	6.751	
0.150	0	18.980	0	26.88	228.73	
	0.262	18.450	0.001836	26.73	139.94	
	2.623	14.780	0.01471	23.29	115.91	
	26.230	4.948	0.04925	19.28	22.28	
	78.67	1.998	0.05965	17.78	6.97	
	262.20	0.6496	0.06440	18.13	2.394	
	524.50	0.3290	0.06551	20.73	1.820	
	1000.00	0.1740	0.6606	27.65	2.284	
	1400.00	0.12465	0.06623	38.51	4.419	

TABLE 4 (continued)

U^{235} enrichment	Atomic ratio H/U^{235}	Density of uranium, gm/cc	Hydrogen atomic density, $\times 10^{24}$ atoms/cc	Critical radius, cm	Critical mass U^{235} , kg	Reflector savings, cm
0.300	0	18.920	0	16.95	114.70	5.11
	1.160	15.150	0.01336	16.33	81.99	4.60
	4.640	9.473	0.03337	16.55	53.32	-
	11.600	5.410	0.04764	17.16	33.95	4.62
	23.200	3.156	0.05558	16.82	18.65	-
	34.800	2.227	0.05884	16.64	12.74	-
	46.420	1.721	0.06062	16.46	9.53	4.34
	116.000	0.721	0.06410	16.17	3.82	3.96
	232.000	0.371	0.06536	16.92	2.232	-
	400.000	0.217	0.06592	18.46	1.696	-
	1000.000	0.08742	0.06636	26.25	1.963	3.49
	1400.000	0.06252	0.06644	35.68	3.506	-
	2000.000	0.04381	0.06652	109.65	71.720	-

the resonance integrals in the multigroup computing scheme. Because of the wide variation in neutron flux anisotropy due to first-order absorption effects, this study was made with the S_4 transport approximation using the anisotropic scattering correction for hydrogen also developed by Bell. The parameter determining the resonance integral for various neutron scattering conditions is $\sigma_p = \sigma_s/N(U^{238})$, macroscopic scattering cross section of the mixture per U^{238} atom. Thus, using σ_s (hydrogen = 20 barns), σ_s (oxygen) = 3.8, the basic relationships used for atomic ratio and atomic density ($\times 10^{24}$ atoms/cc) were:

$$N(H_2/N(U^{238})) = 0.02282 \sigma_p$$

$$H/U = \frac{2 \times 0.02282 \sigma_p}{1 + f}$$

$$N(U) = \frac{0.048}{1 + 0.720 H/U}$$

where $f = N(U^{235})/N(U)$ and N is the number of atoms per cubic centimeter ($\times 10^{24}$).

In addition to critical radius as a function of total uranium density (kg/l) (Fig. 15), the critical mass as a function of H/U^{235} atomic ratio is shown in Fig. 16, and the structure of the asymptotes for infinite size of uranium enrichment as a function of H/U^{235} atomic ratio in Table 5 and Fig. 17. Note that both of these last figures show 1.0% enrichment to be the least possible value for a critical reactor using U^{235} in H_2O . Figure 18 is for UF_6 in water (because of its importance in reactor safety studies). The others are all $U-H_2O$ mixtures, with corrections as

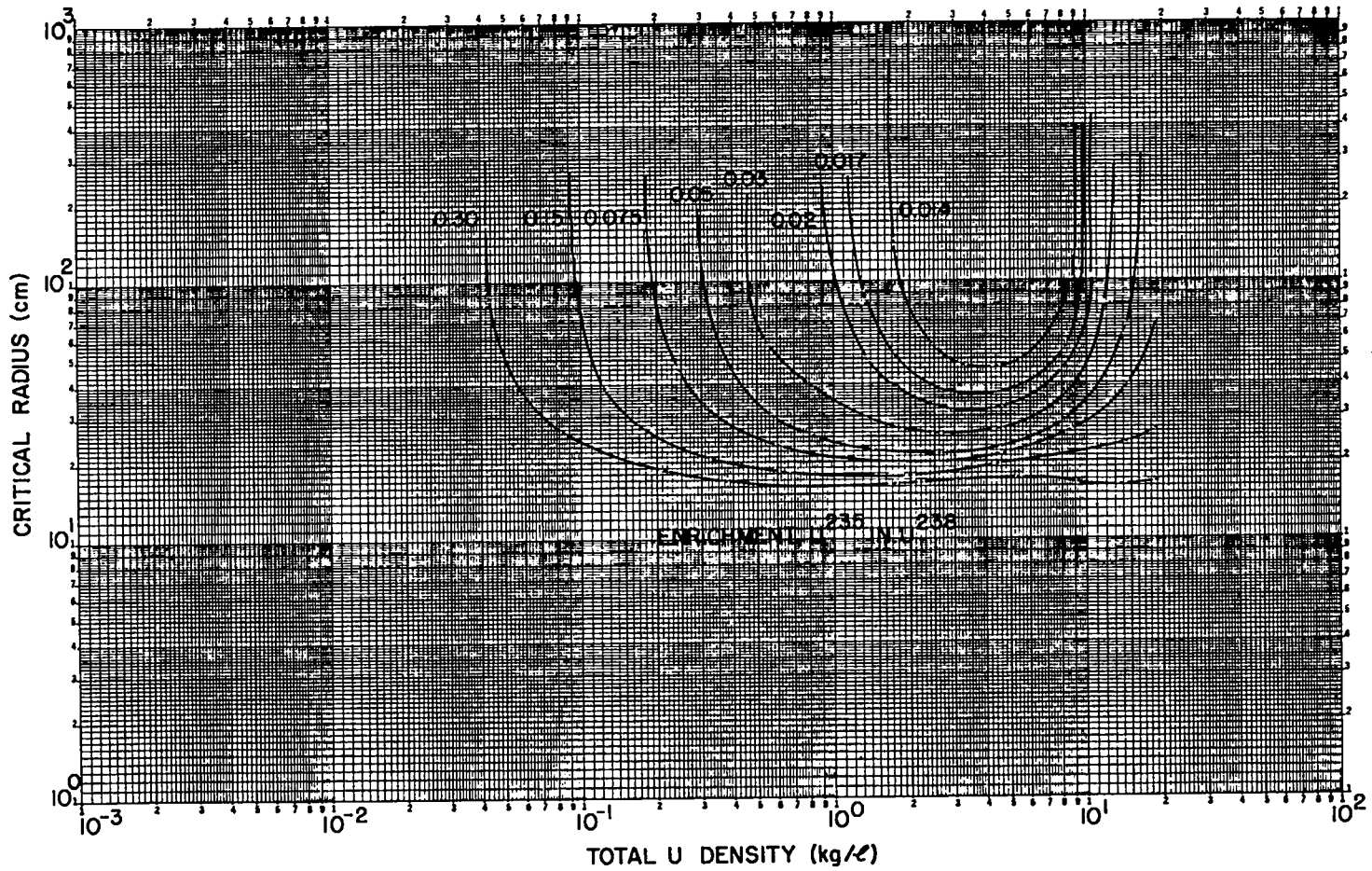


Fig. 15. Low-enrichment U^{235} critical radius for spheres as a function of total uranium density in metal-water solutions.

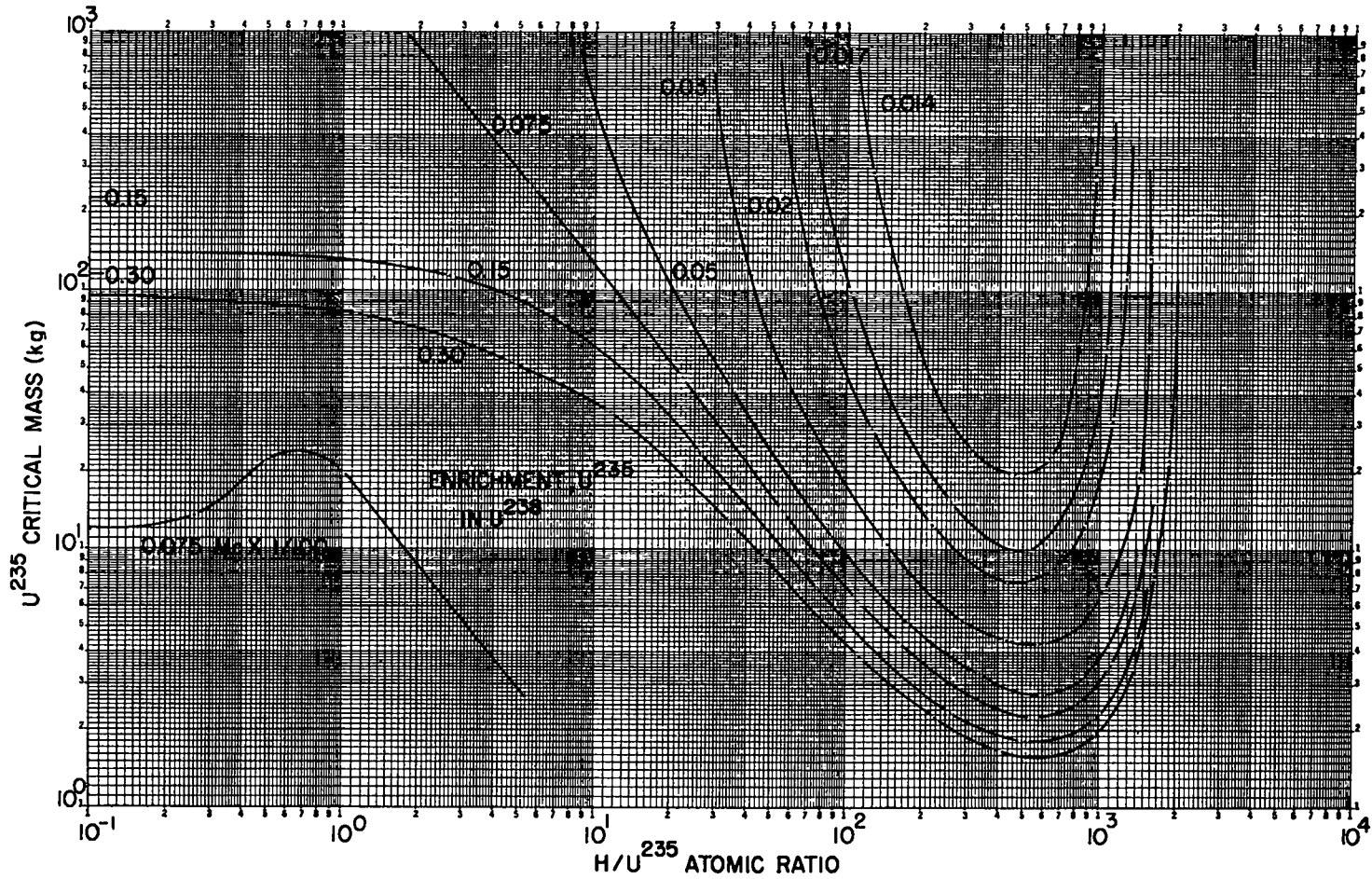


Fig. 16. Low-enrichment U^{235} critical mass for spheres as a function of H/ U^{235} atomic ratio in metal-water solutions.

TABLE 5. INFINITE CRITICAL ASSEMBLIES

U in H ₂ O		UF ₆ in H ₂ O	
Atomic ratio H/U ²³⁵	Enrichment, atom percent	Atomic ratio H/U ²³⁵	Enrichment, atom percent
0	5.77	0	7.65
0.67	6.39	2.1	6.38
11.1	3.95	21	3.82
37.5	2.38	142	1.29
150	1.202	178	1.00
364	0.998		
993	1.34		
1134	1.60		
1381	1.96		
1567	2.84		
1929	9.39		
2040	22.2		
2093	43.2		

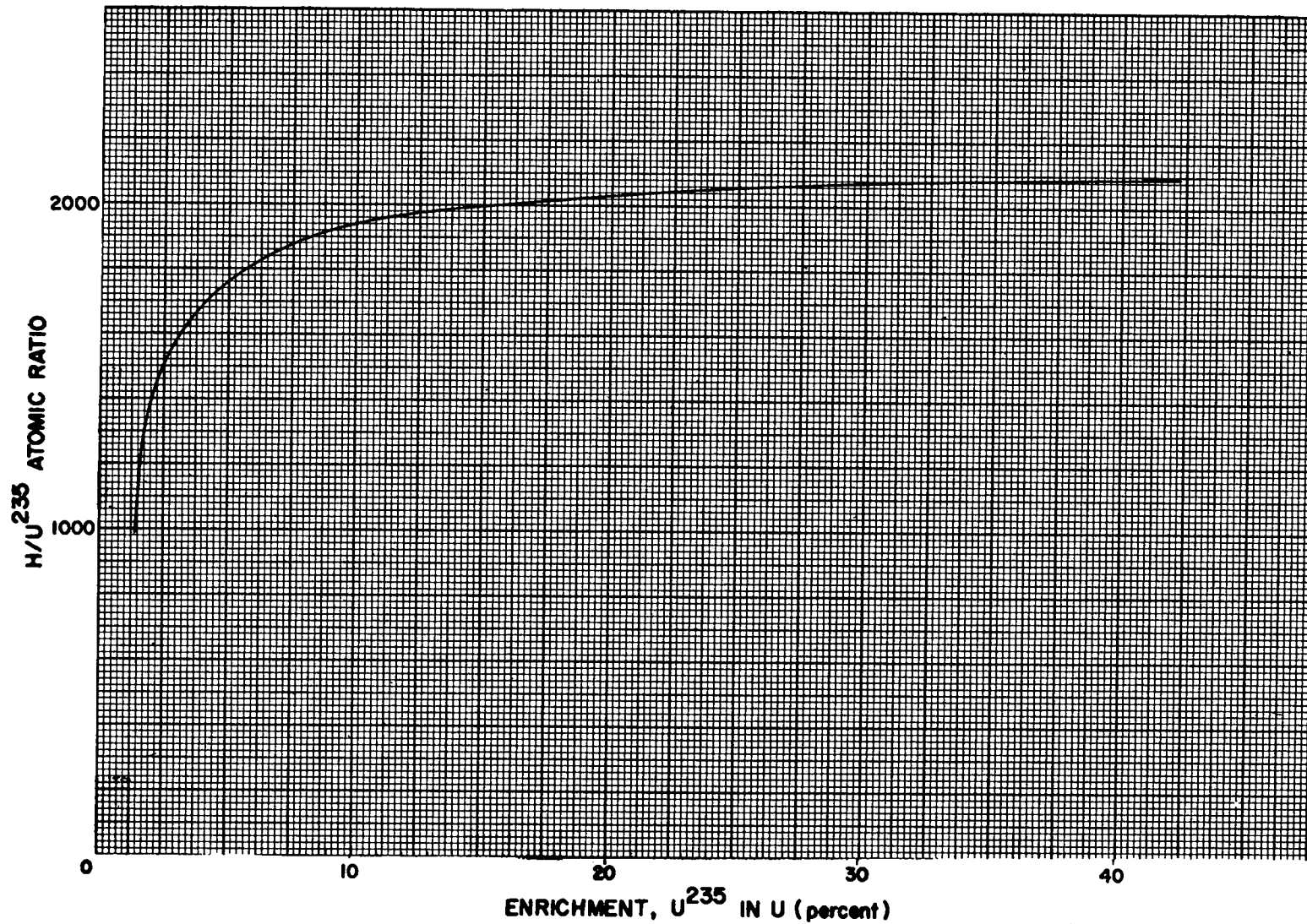


Fig. 17. H/U^{235} atomic ratio as a function of enrichment for just-critical infinite metal-water solutions.

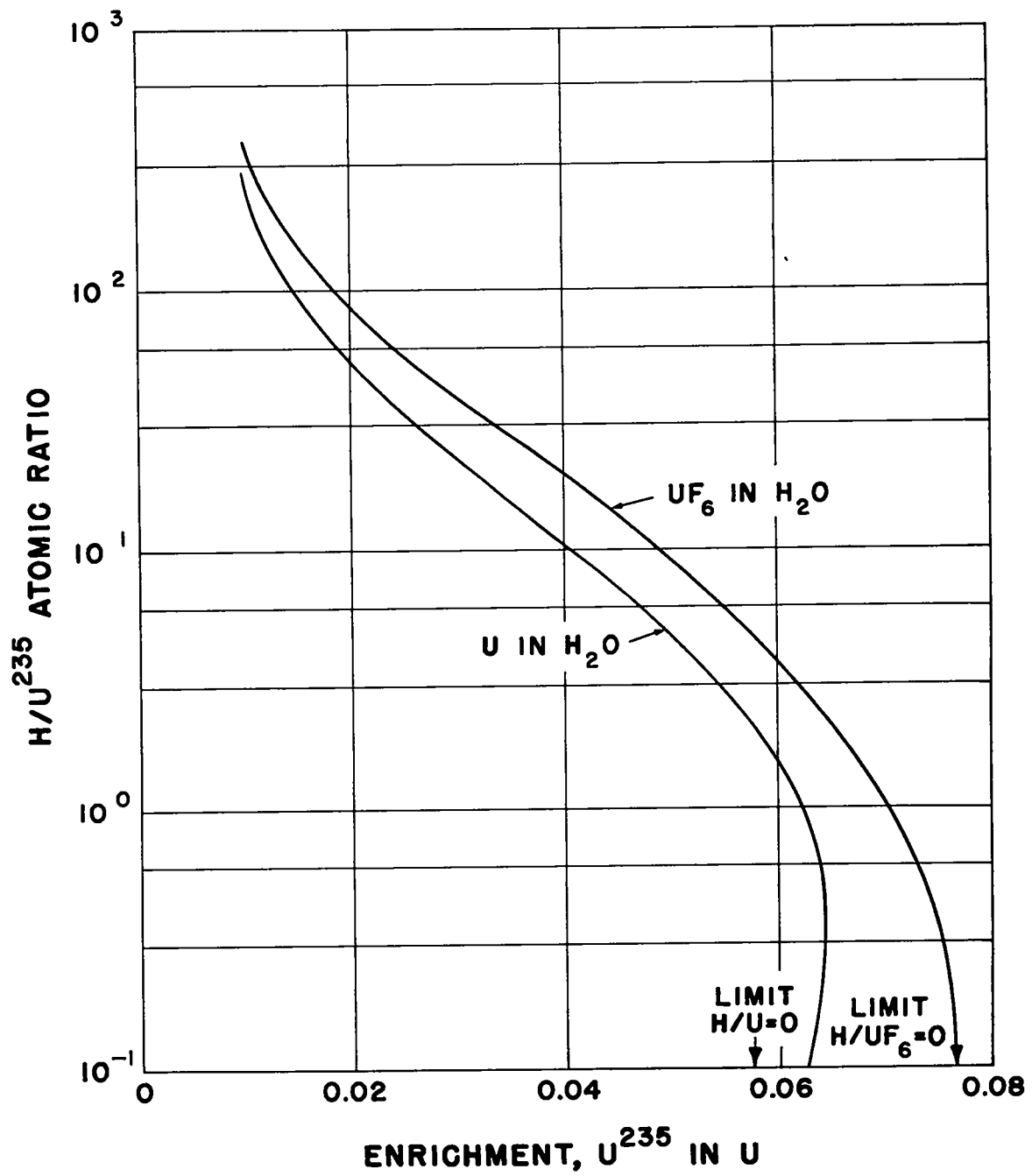


Fig. 18. H/U^{235} atomic ratio as a function of enrichment for just-critical, infinite, U, and UF_6 -water solutions.

shown above for mutual volume displacement. Figure 19 shows the value of an infinite water reflector in reducing core size for a few points.

Supplementary results for UF_4 in paraffin and UO_2F_2 in water are of interest for reactor safety experimental studies.* Results of analyses of the same type as those shown just above, but with

$$N(U) = \frac{0.01298}{1 + 0.318K} \text{ for } UF_4 \text{ in paraffin}$$

and

$$N(U) = \frac{0.01752}{1 + 0.519K} \text{ for } UO_2F_2 \text{ in water}$$

where K is the moderator (CH_2 or H_2O) to uranium number density ratio, are shown in Table 6 and Figs. 20 and 21. Here the percentage values refer to weight percent U^{235} in U^{235} plus U^{238} .

The effect of an infinite paraffin reflector on a spherical core was determined for the 2% enrichment set of calculations. The results are shown in Fig. 22 as a function of total uranium density (g/cc) for direct comparison with the experimental values.

TEMPERATURE EFFECTS IN GRAPHITE

The parametric indication of the effect of temperature and C/U^{235} atomic ratio on graphite is shown in Table 7. There is clearly a small effect for small C/U^{235} ratios because of the small thermal flux and a large effect for high C/U^{235} atomic ratio systems, in which the thermal

*These figures supplement work by Mihalczko and Cronin of ORNL.

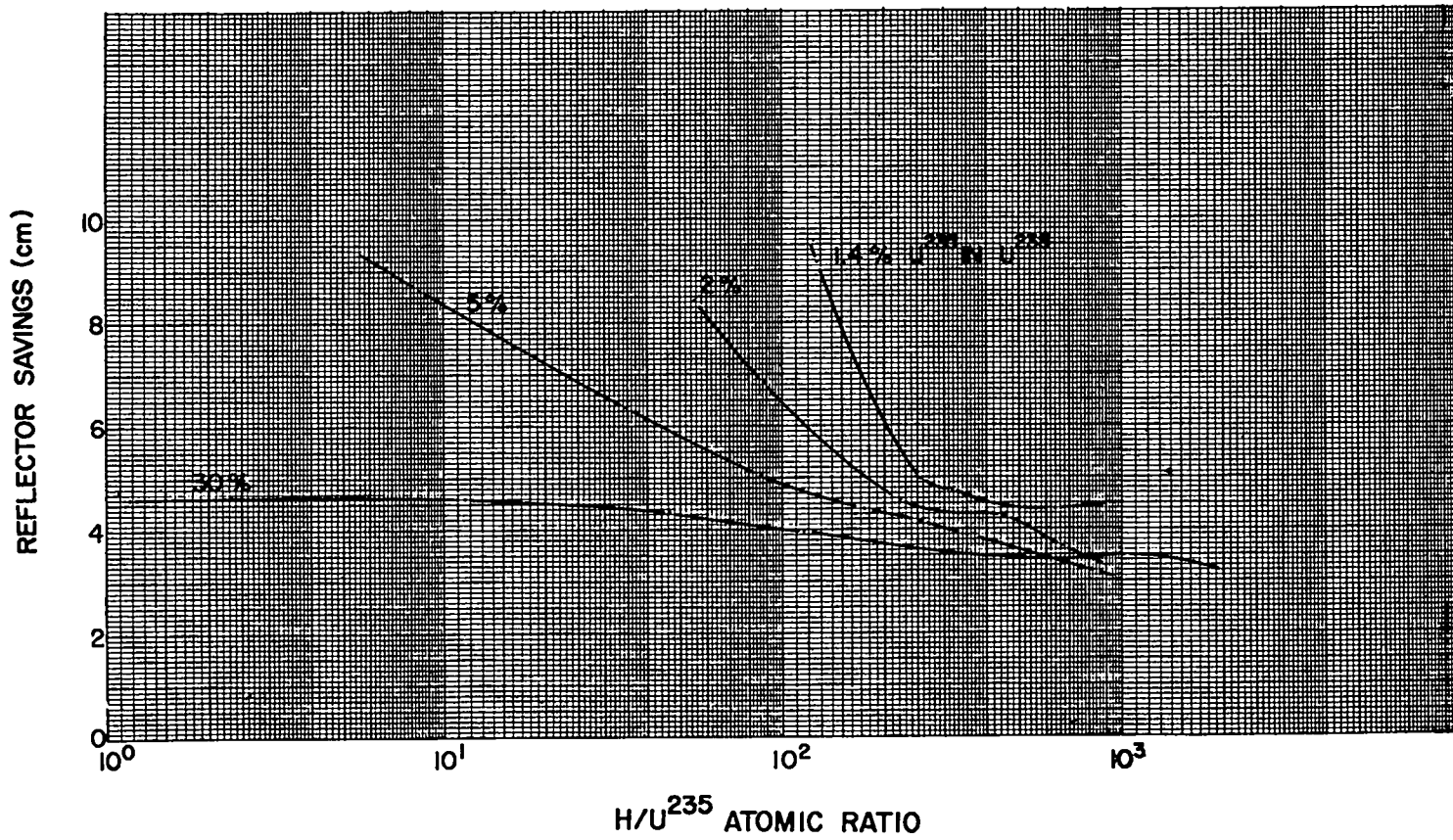


Fig. 19. Reflector savings vs H/U^{235} atomic ratio for various enrichments of spherical, metal and water reactors reflected by water.

TABLE 6. CRITICAL RADII FOR LOW-ENRICHMENT URANIUM

	Total Uranium Density, g/cc				
UF_4 in Paraffin	4.00	3.28	2.124	1.339	0.635
2%, critical radius, cm	73.52	45.2	35.0	46.6	Large
3%, critical radius, cm	54.23	38.0	27.6	29.1	63.0
5%, critical radius, cm	43.82	31.7	23.3	22.4	27.3
	Total Uranium Density, g/cc				
UO_2F_2 in water	4.73	3.60	2.093	1.233	0.552
2%, critical radius, cm	66.08	42.4	36.9	52.1	Large
5%, critical radius, cm	37.71	29.6	24.5	24.5	31.7

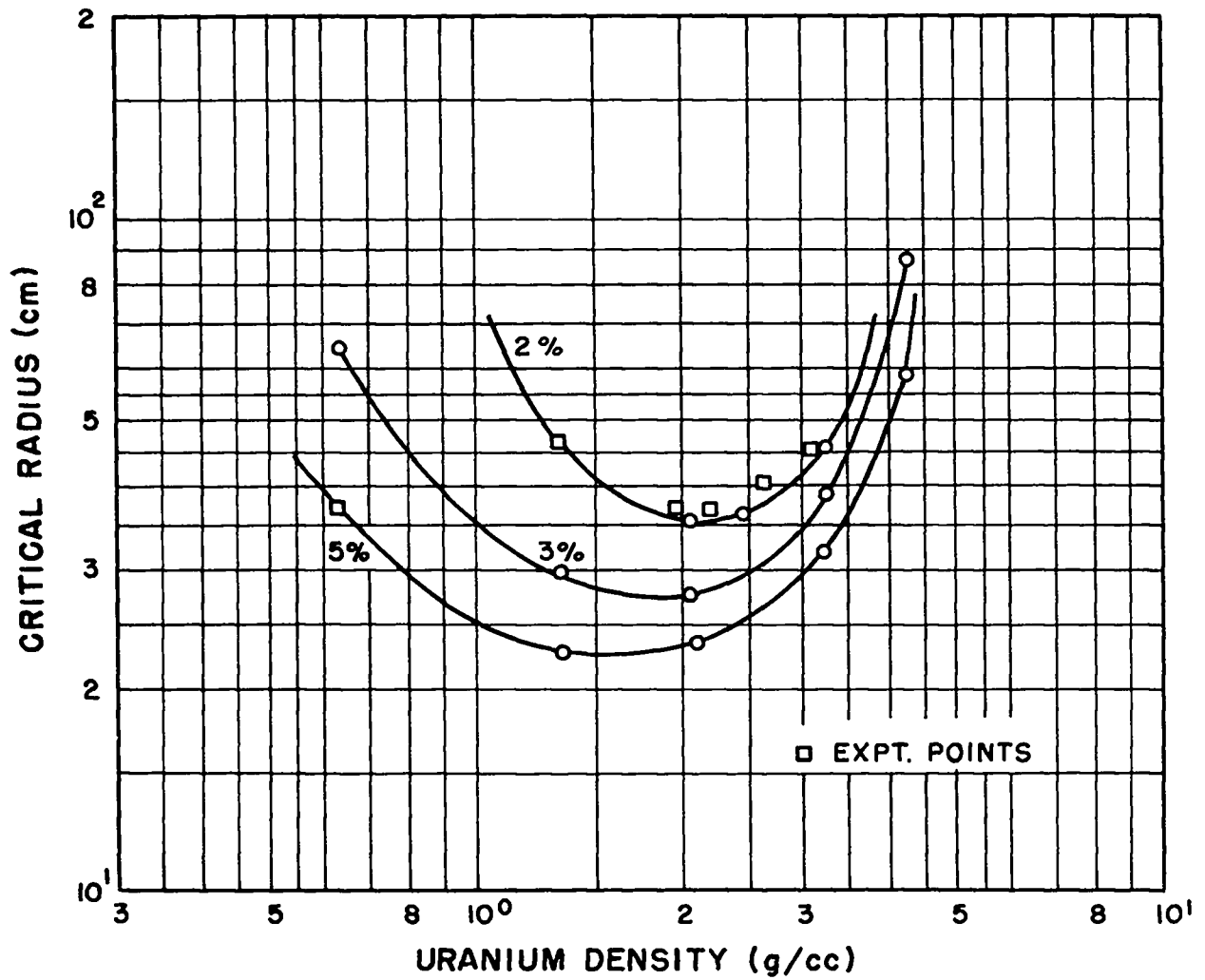


Fig. 20. Critical radius as a function of uranium density in paraffin for 2, 3, and 5% enrichment in U^{235} .

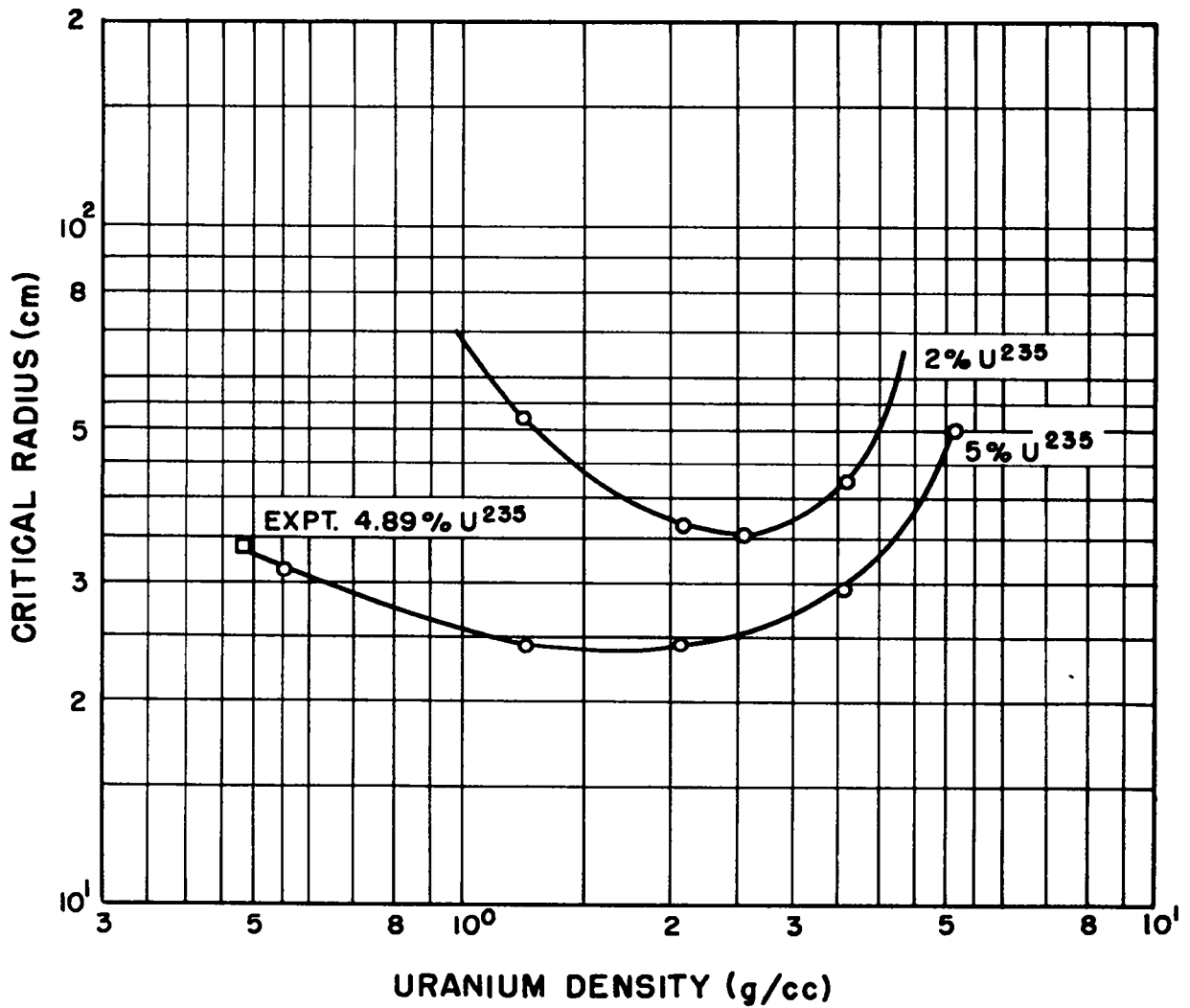


Fig. 21. Critical radius as a function of uranium density in water for 2 and 5% enrichment in U²³⁵.

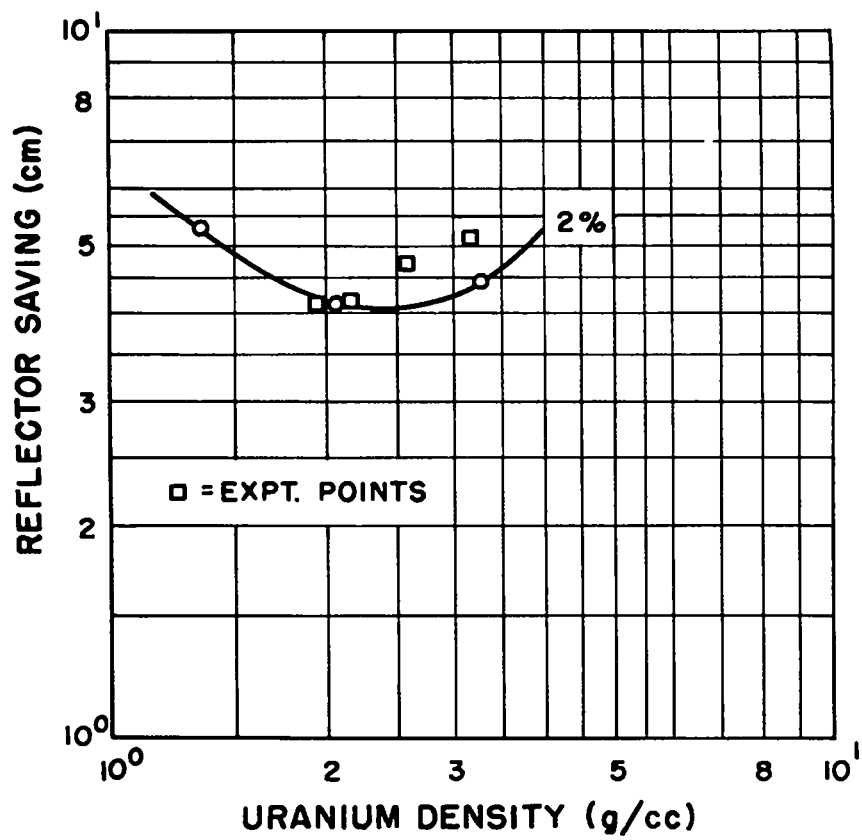


Fig. 22. Reflector savings as a function of uranium density in paraffin for 2% enrichment in U^{235} .

TABLE 7. TEMPERATURE EFFECTS ON CRITICALITY IN CARBON

Atomic ratio ¹³ U ²³⁵	301	603	1206	2355
Temperature, ev		k_{eff}		
0.025	1.007	0.991	0.997	1.004
0.056	1.005	0.986	0.983	0.973
0.100	1.004	0.975	0.954	0.916
0.152	1.002	0.971	0.940	0.882
0.194	1.001	0.968	0.931	0.867
Radii, cm	63.48	64.66	66.62	69.36
Temperature, ev	Critical radii, cm; same C/U ratios			
0.025	63.5	64.7	66.6	69.4
0.056	63.6	64.9	67.5	71.2
0.100	63.7	65.6	69.2	74.6
0.152	63.8	65.8	70.0	76.6
0.194	63.9	66.0	70.6	78.1

group contributes most of the fissions. Figure 23 shows the same thing in a more striking way, emphasizing the very small error in the results of analyses of high-temperature graphite reactors.

REFLECTOR-MODERATED REACTORS

The purpose of this section is to establish the minimum critical mass and concentration of U^{235} in a wide variety of cavity (or reflector-moderated) reactors. These are of current interest because of the essentially unlimited temperatures possible with the fission process in the gaseous state. The parameters are geometry (spheres and cylinders) and radii, with some attention given to neutron absorption by structural materials.

The relative value of D_2O , Be, and C as reflectors of a U^{235} gas-filled cavity is shown in Fig. 24. Note that Be and D_2O are equivalent for thicknesses less than 50 cm, and that the use of D_2O in a thick layer is consistent with the lowest critical concentration of U^{235} . The dependence of critical core (cavity) radius on the reflector is shown in Fig. 25. Note the minimum in the critical mass of U^{235} gas with a Be reflector.

The limits of criticality of U^{235} gas are explored in Fig. 26 to 28, which show critical mass and atomic density of U^{235} for large spherical and cylindrical cavities reflected by D_2O . Note the small value of increased D_2O thickness and the relatively large effect of a structural layer at the cavity wall.

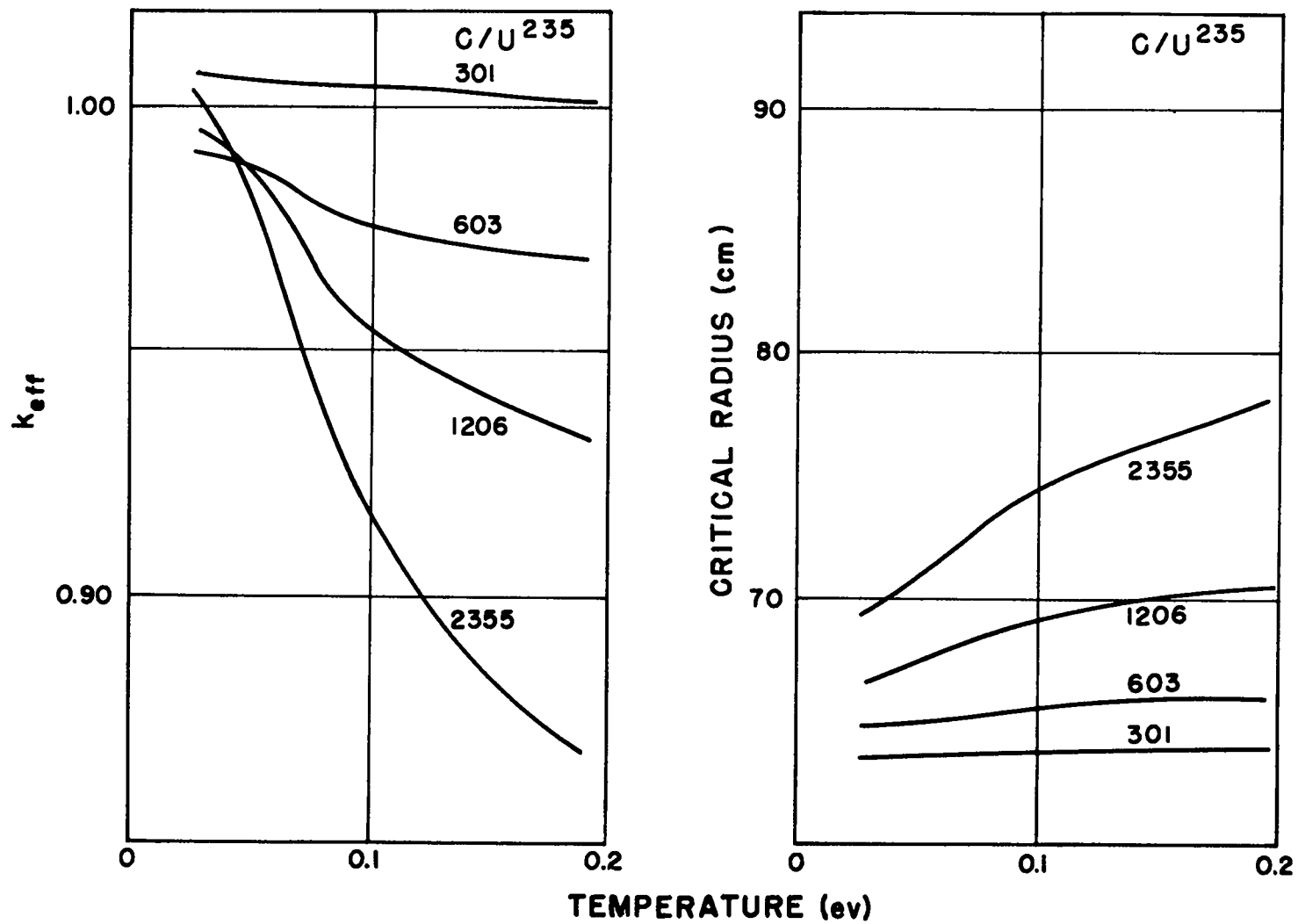


Fig. 23. The effect of reactor temperature on k_{eff} and critical radius in a graphite reactor.

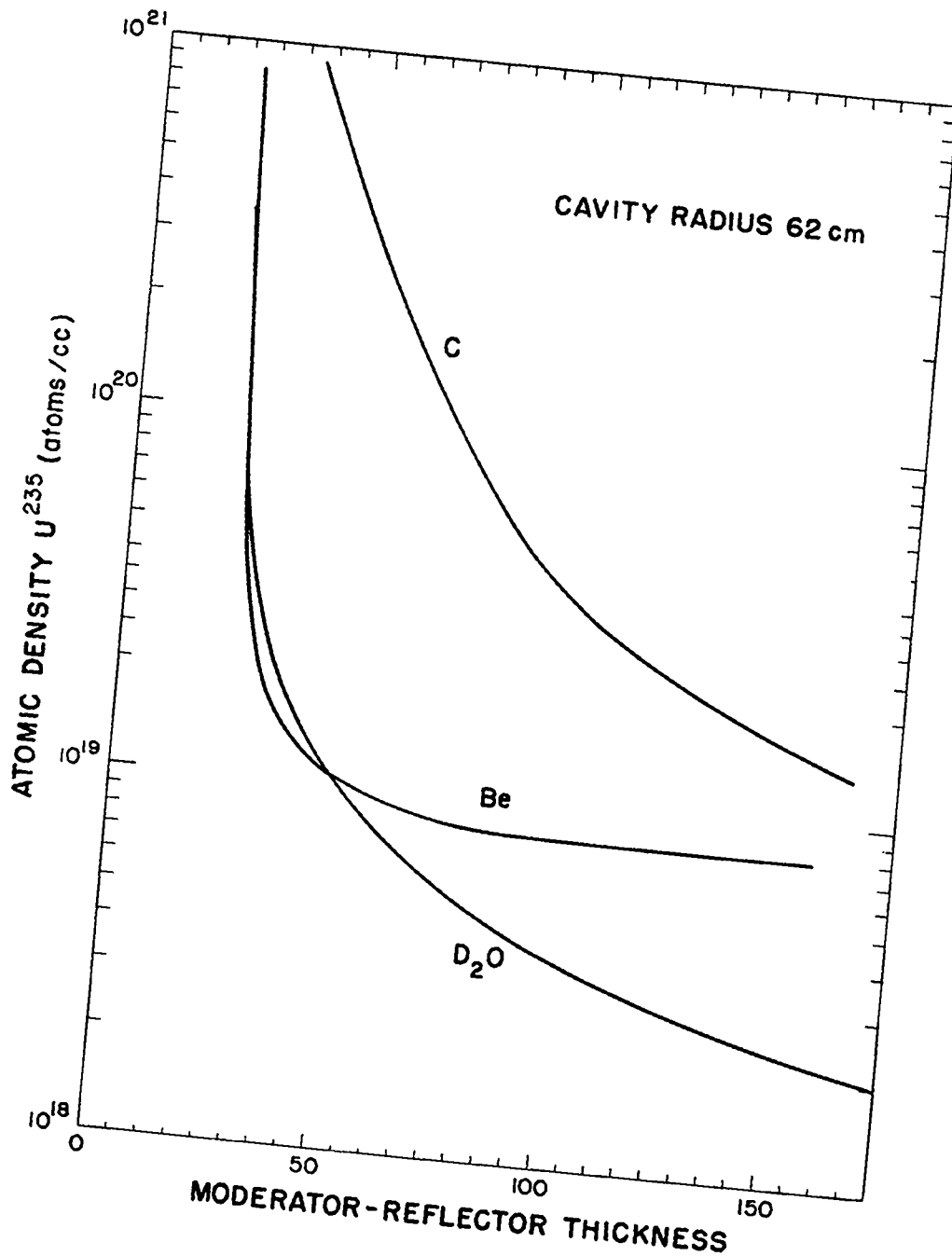


Fig. 24. Critical concentration of U^{235} gas as a function of D_2O , Be, and C-reflector thickness.

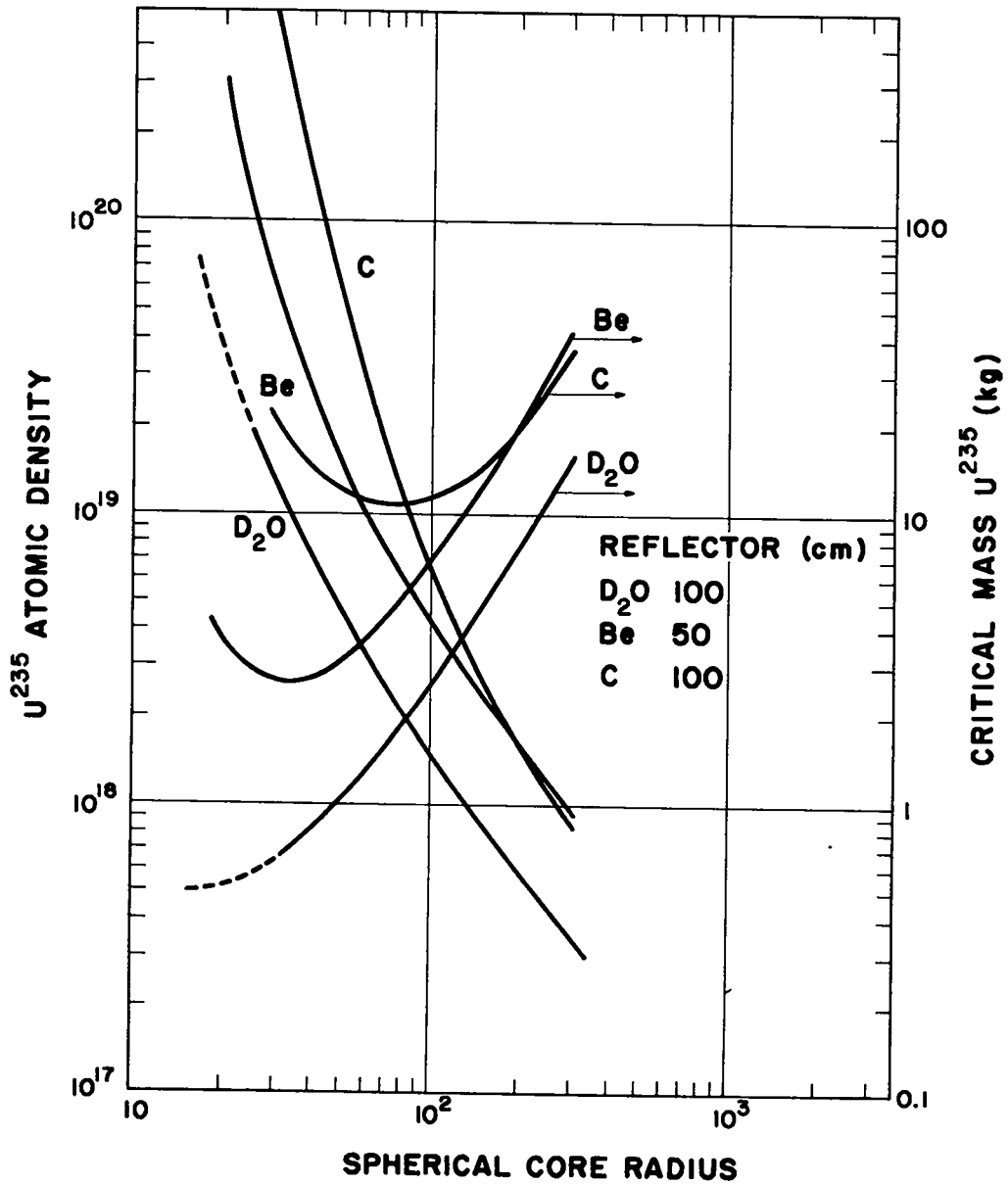


Fig. 25. Critical concentration of U²³⁵ gas as a function of D₂O, Be, and C-reflected core radius.

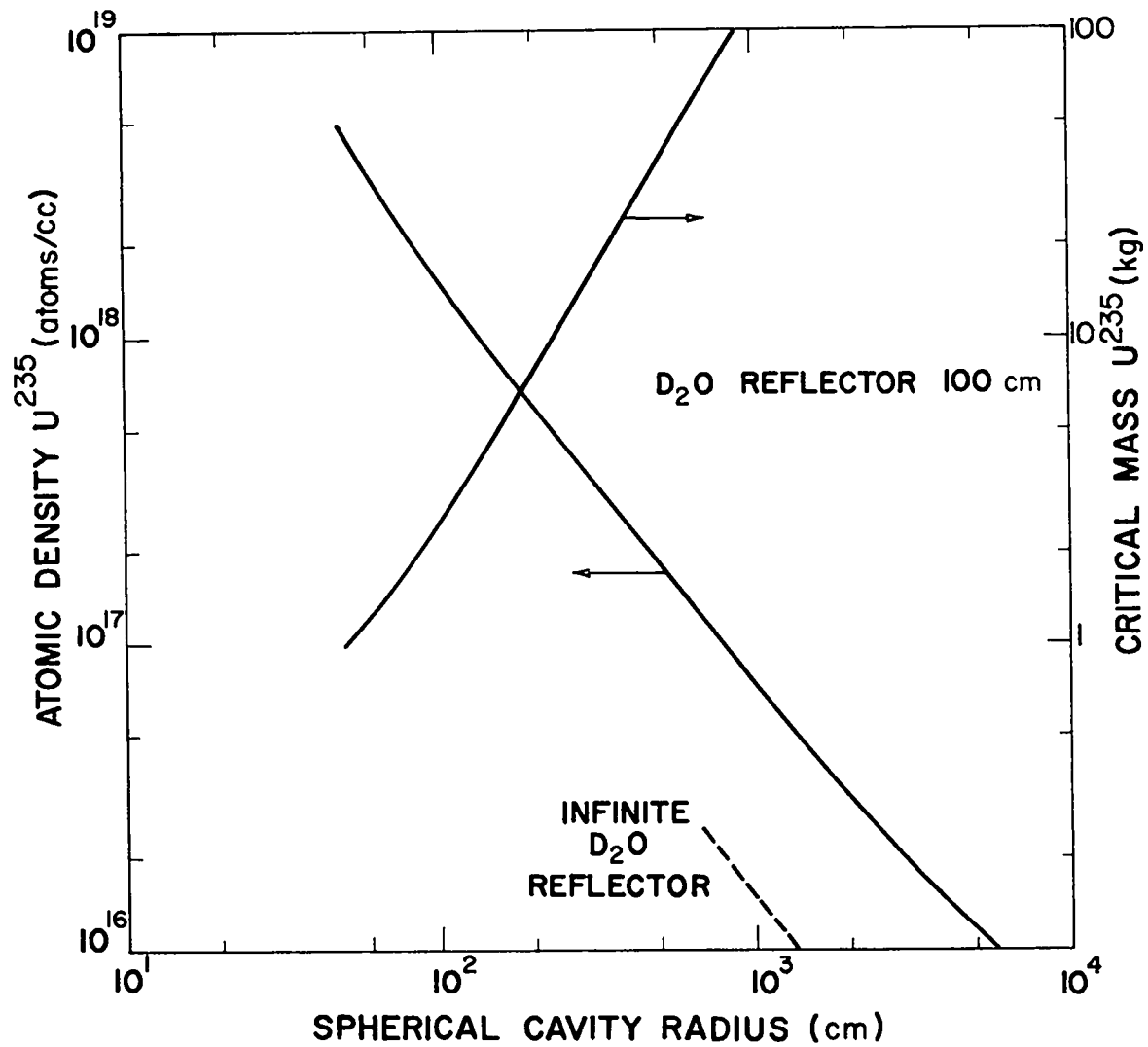


Fig. 26. Critical concentration and mass of U^{235} gas vs spherical core radius for a cavity reflected by 100 cm of D_2O .

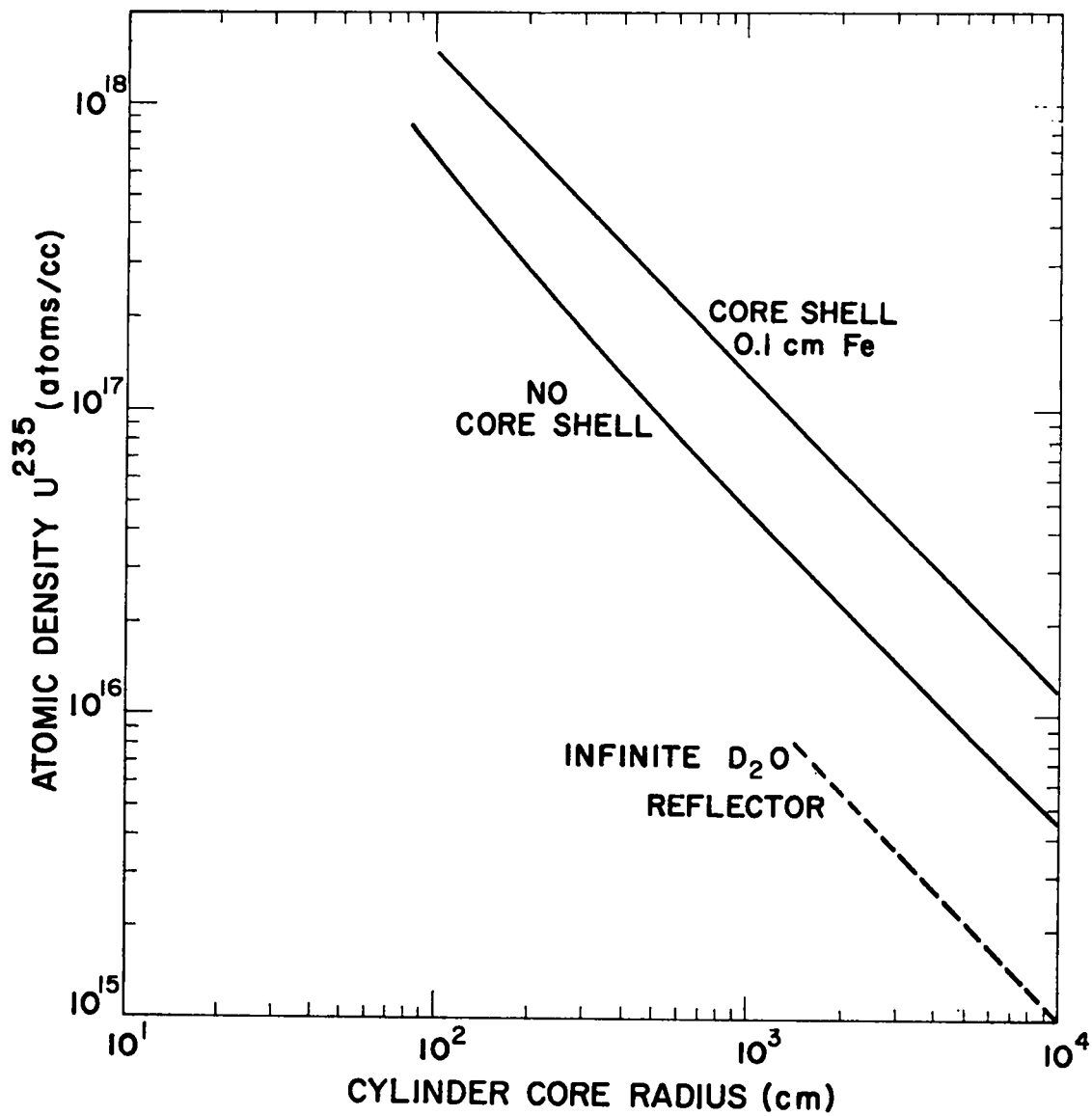


Fig. 27. Critical concentration of U²³⁵ gas vs cylinder core radius for a cavity reflected by 100 cm of D₂O.

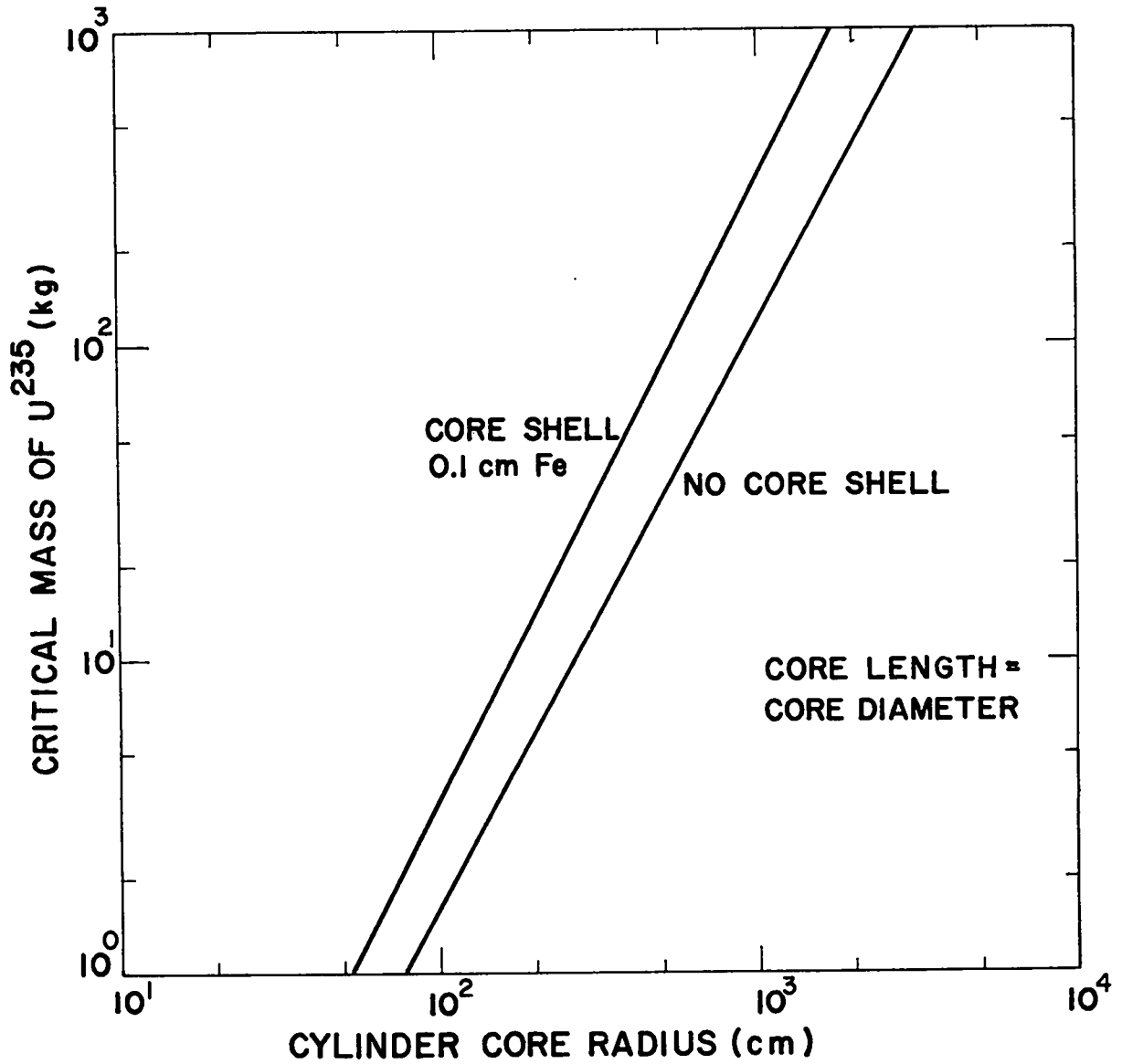


Fig. 28. Critical U^{235} mass vs cylinder core radius for a cavity reflected by 100 cm D_2O .

An interesting extrapolation of the cavity reactor concept is shown in Fig. 29 for a cylinder reflected by 100 cm (0.1 cm Fe lining the cavity wall) D_2O but with the U^{235} restrained to smaller radii than the radius of the core (dotted line). (The $S_4 - S_{16}$ approximations were required for these curves.) Figure 30 shows the critical-mass dependence for this set for cylinder length equals diameter. Neither critical mass nor atomic density of U^{235} is a strong function of gas radius until the area reduction is very large.

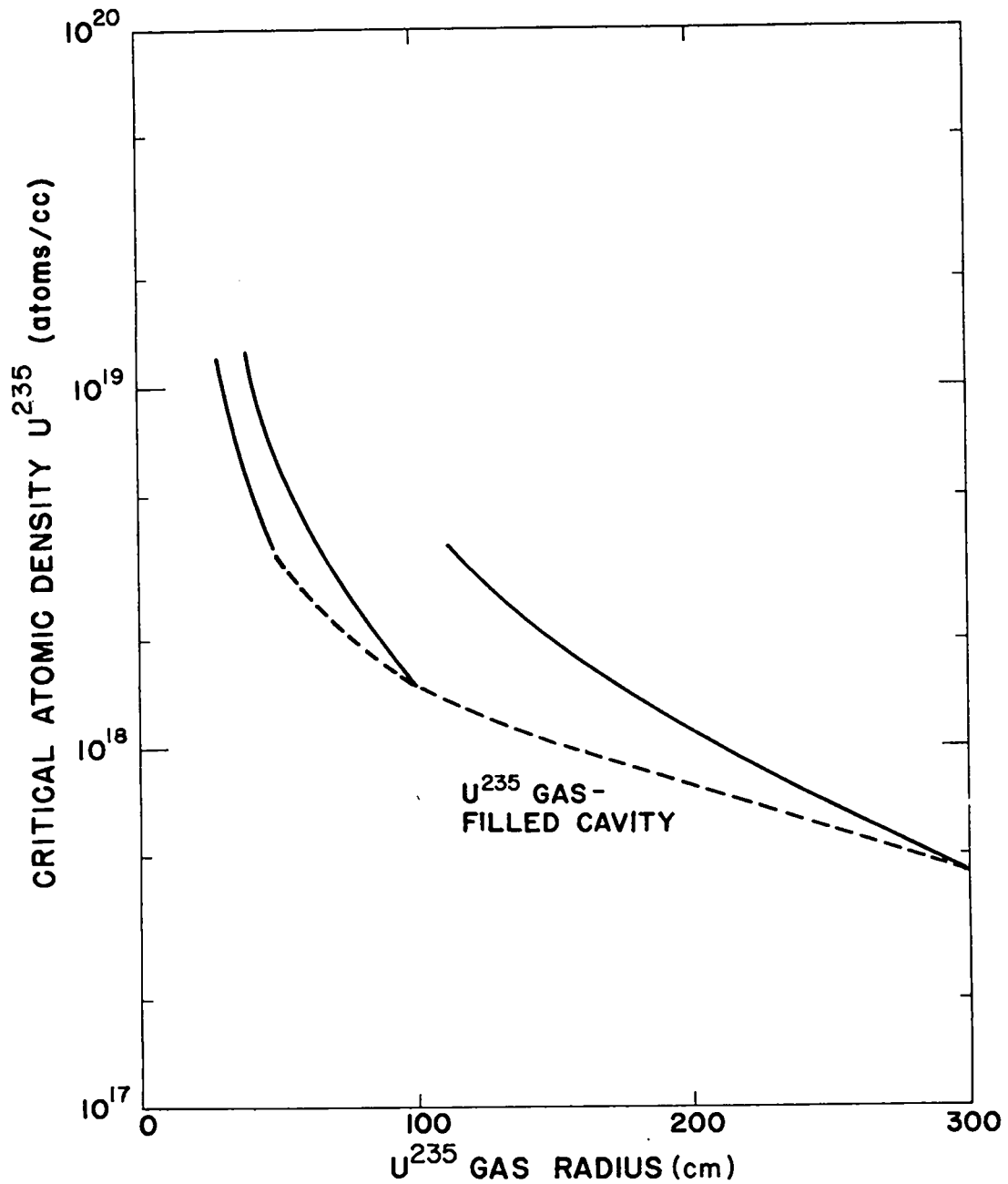


Fig. 29. Critical concentration of U^{235} gas vs axial cylindrical column of radius for a cavity of constant radius reflected by 100 cm D_2O .

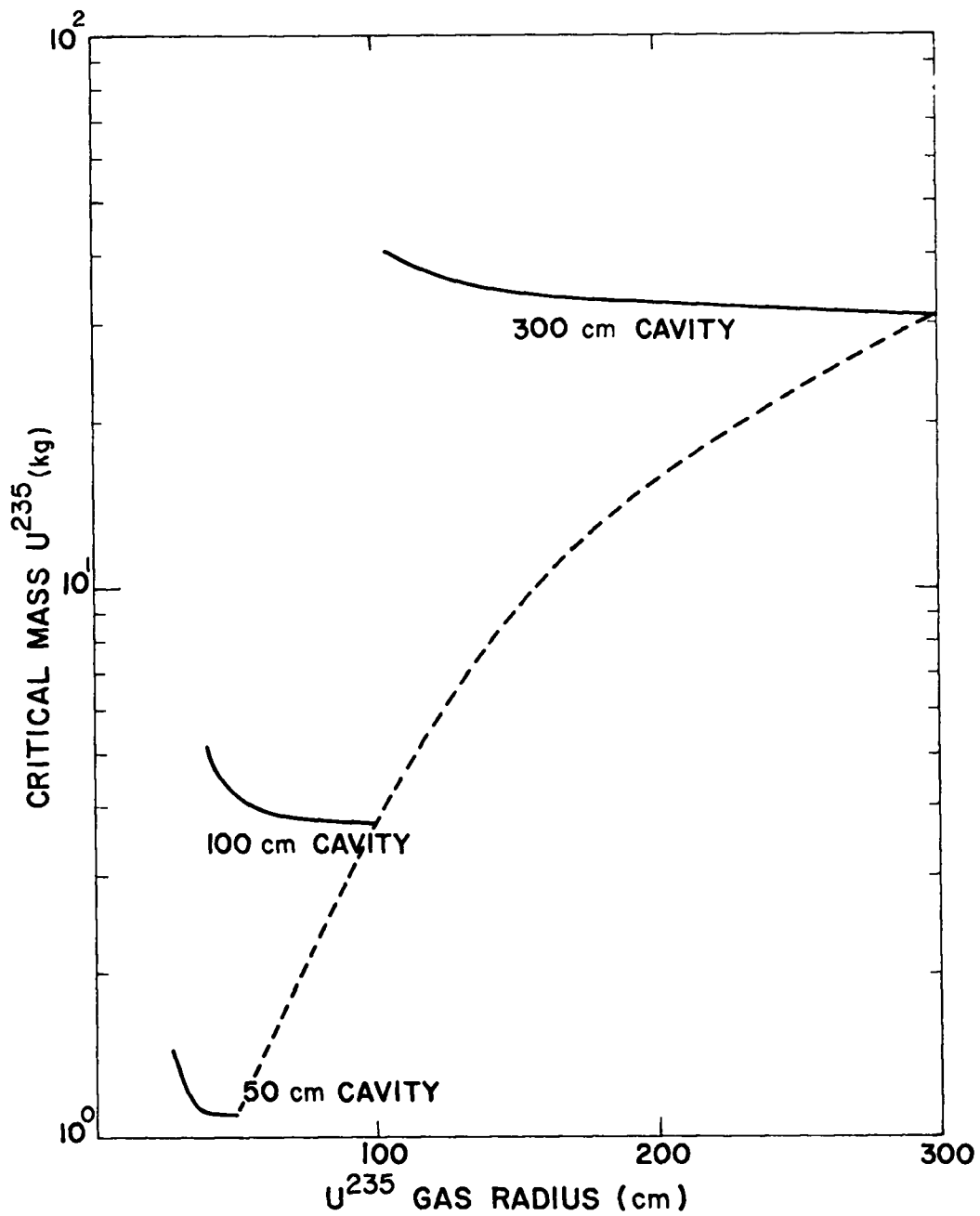


Fig. 30. Critical U²³⁵ mass vs axial cylindrical column radius for a cavity of constant radius reflected by 100 cm D₂O.

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